

# Scheduling with Collaborative Mobile Charge Problem with Time Windows in 2-D Wireless Sensor Networks

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**Abstract:** Recent breakthroughs in wireless energy transfer and rechargeable lithium batteries emerge the problem of mobile charge. Mobile charge describes a fleet of mobile chargers deliver energy to sensor nodes periodically and maintains the perpetual operation of the network. However, different energy consuming rate of each sensor node imposes a great challenge in scheduling mobile chargers. A follow-up study based on [1] and [2], we take into account the different energy consuming rate, to give the definition of time windows for the sensor nodes. In this paper, we propose a Collaborative Mobile Charge problem with Time Windows (CMCTW) and employ a Vehicle Routing Problem with Time Windows (VRPTW) model to solve it. Firstly, for determining the number of the mobile chargers and other important parameters, we investigate the quantitative relationship between problem model and solution model. Moreover, for solving the problem of multiple mobile chargers with different routing paths, we propose to transform the multiple routing problems into a single routing problem, by duplicating the sink into multiple virtual sinks. To improve the routing path, we propose a local optimization algorithm by considering the collaborative charging among the mobile chargers. The experimental results show that our collaborative scheduling compares with the art-of-state algorithm is significantly better in solution stable and accurate.

**Keywords:** Mobile charging with time window, Collaborative schedule algorithm, Wireless sensor networks

## 1. Introduction

Recent several decades, with flourishing of researches in Wireless Sensor Networks (WSNs), the applications of WSNs spread from American military to many living fields, such as home automation [3], forest fire detection [4], structural health monitoring [5], and rainfed agriculture [6]. However, the limited energy of the sensor node becomes one of the most critical impediments, which stunts the development of WSNs. Some researches on energy conservation [7] and energy harvesting [8] have shown great promise in addressing this issue. On the other hand, the energy conservation can prolong the lifetime of the sensor. However, the technology for obtaining energy from the environment is still unstable, unpredictable and uncontrollable.

Recent breakthroughs in wireless energy transfer and rechargeable lithium batteries provide a promising support for the energy problem of WSNs. Experimental demonstration in [9] shows the energy can be efficiently transmitted between magnetically resonant objects without any conduction.

Rechargeable lithium batteries own high energy densities and high charge capabilities have shown in [10].

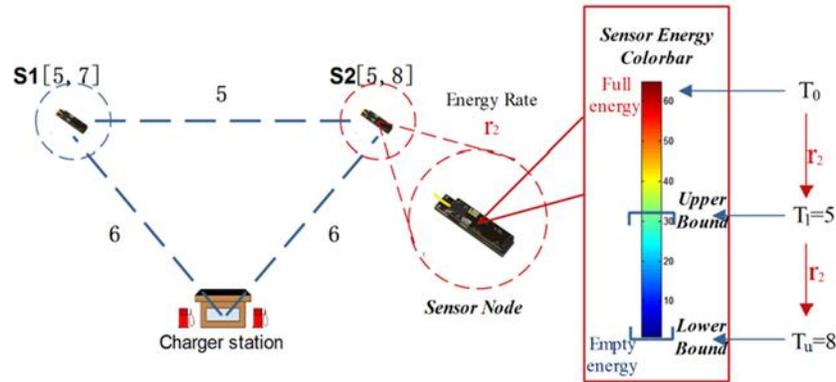


Fig. 1. Energy bound and Time Windows

Supporting by these enabling technologies, some studies envisioned employing a mobile vehicle which carried a high volume battery as a mobile charger to periodically deliver energy to the sensor nodes. JieWu et al focused their attention on collaborative and coverage of mobile charge [11], and the works in [12], [13], [14] draw their attentions on 1-dimension state and simply extend to 2-dimension. These works focus on the maximum total vacation time of mobile chargers. Y.Y ang et al combine the mobile charge problem with data gathering and employ technic such as WerMDG [15],NETWRAP [16] and so on, keeping the minimize delay and tour path. ThomasHou et al [17] proposed an optimization model to give the vocation time and stop point of mobile charger under a certain path. One utility function was employed as a goal function.

However, most of them assumed that a sensor node can be charged as long as its energy consumed. It is obvious that charging a sensor node one time with the energy of  $B/2$  energy ( $B$  is the sensor capacity) is more effective than charging a sensor twice with the energy of  $B/4$ . Thus, it is reasonable to give a energy upper bound which means it is unnecessary to charged when a sensor's residual energy larger than this upper bound. As shown in Fig. 1, sensor node and its energy described in right hand. In red box, the color bar imply the sensor energy while red color means full energy and blue color means empty energy for sensor node. Upper bound and lower bound of sensor energy are two numeric can conduct upper and lower bound of time windows. For example, sensor node S2 with energy rate  $r_2$  working from  $T_0$ . As sensor energy reach to energy upper bound, time  $T_1 = 5$  is the lower bound of sensor S2 time windows. Then, time  $T_u = 8$  is the upper bound of sensor S2 time windows.

We give the time windows for each sensor node such as shown in Fig. 1 S1 and S2. When the velocity of the mobile charger is fixed, it moves from the charger station to the sensor nodes s1 and s2, costing 6 time units respectively. Then, it moves from s1 to s2 and costs 5 time unit. Here, [5,7],[5,8] are the time windows of the sensor node s1 and s2 respectively.  $8 - 5 < 5$ , means the upper bound of s2 (8 time units) minus lower bound of s1 is less than the time cost from s1 to s2. It means that s1, s2 needn't be charged by one charger during their time windows at the same time. In [12], it considered the battery capacity of chargers and gave his collaboration algorithm H $\eta$ ClusterCharging( $\beta$ ) to deal with the schedule problem of the mobile charge in 1-dimension and simply extended it to 2-dimension. But, it ignores the state above.

Based on the observation above, we describe the problem in details as follows. A WSN, all sensor nodes in it own their time windows. Scheduling a fleet of mobile chargers with the same battery capacity and velocity(their velocity is fixed) to charge all sensor nodes in their time windows. The objective is to minimize the total tour length or any other goals. Fig. 2 shows a solution of our problem. A fleet vehicle with three mobile chargers start from charger station to charge all sensors and come back to charger station when finish this task. If charger comes early, it should be wait until energy consumed under a threshold.

In this paper, we propose a collaborative mobile charge problem with time windows(CMCTW) and employ Vehicle Routing Problem with Time Windows (VRPTW) model to solve it. We aim at optimizing the total energy level after charging the WSNs one time. To this end, we add virtual charger station based on the number

of the chargers, and then transform the problem into a Hamilton cycle problem. The number of the chargers can be obtained based on the quantitative relationship of charger's property parameter. Then, we improve the Hamilton cycle via specific local optimization and collaboration among mobile chargers. The contributions of this paper are summarized as follows:

1. We introduce the time windows in this problem, and transform this novel problem into vehicle routing problem with time windows (VRPTW), which is classical problem.
2. We give several quantitative relationships between problem model and solution model to solving this problem more reasonable and accurate.
3. We utilize the virtual charger stations to transform the problem into Hamilton cycle problem

The rest our paper is organized as follows: Some related works are presented in section II. The problem formulation is given in section III. In Section IV, we investigate the quantitative relationships between the problem model and the solution model. Section V presents our schedule scheme, and Section VI evaluates it through simulations. Finally, Section VIII summarizes the paper and outlines the research perspectives.

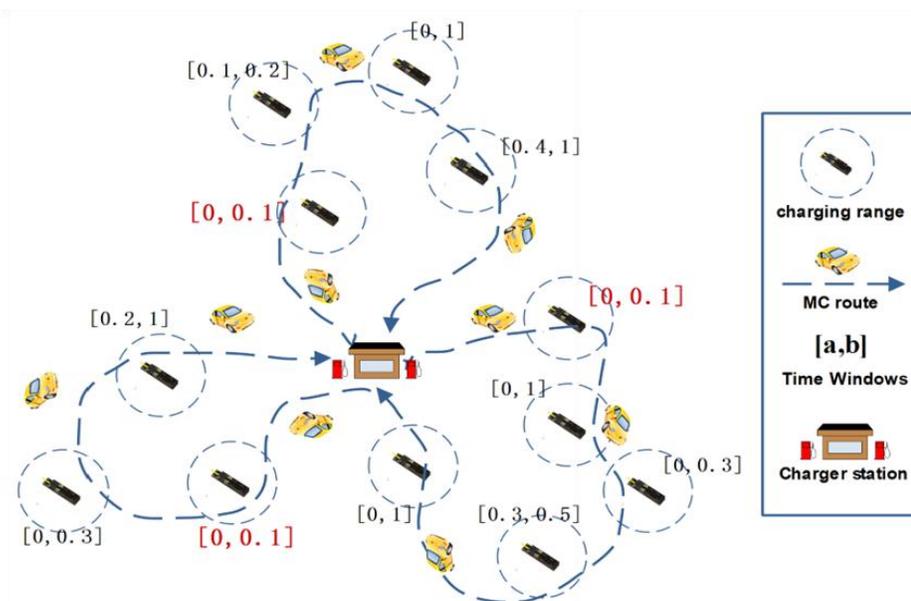


Fig. 2. A solution of our problem

## 2. Related Work

Many prior works in prolong the lifetime of WSNs that supported and inspired our basic idea, especially this paper utilizes the research foundation of [1] and [2], and will consider more factors to further study the problem of mobile charge. We describe a part of these related efforts in this section.

**Existing energy transfer algorithm.** Peng et al. [18] Li et al. [19] focused on maximizing network lifetime through finding an optimal charging sequence. Shi et al. [20] assumed that the mobile charger has unbounded energy, and investigated the problem of periodically charging sensors to maximize the ratio of the chargers vacation time (time spent at the home service station) over the cycle time. They further considered the multi-node simultaneous charging scenario [21], [22]. Fu et al. [23] focus on minimizing the total delay of replenishing all sensor nodes in a network. JieWu et al focus their attention on collaborative and coverage of mobile charge [11], and the works in [12], [13], [14] focus on 1-dimension statue and simply extend to 2-dimension. These works focus on the maximum total vacation time of mobile chargers. Y.Y ang et al combine the mobile charge problem with data gathering and employ technic such as WerMDG [15], NETWRAP [16] and so on, keeping the

minimize delay and tour path. ThomasHou et al [17] propose an optimization model to give the vocation time and stop point of mobile charger under a certain path. One utility function is employed as goal function.

Before we give our problem formulation, we firstly to give the general problem definition of mobile charging problem [12]. In general problem, they define the problem model as WSNs model with parameter  $X, Y, B, T$ , and define the solution model as charge model with parameter  $(P, c, v, \eta_1, \eta_2)$ . Where  $X$  and  $Y$  combine the location of node  $s$ ,  $(x_i, y_i | x_i \in X, y_i \in Y)$ .  $B$  is the sensor capacity set and  $T$  is the recharging cycle set of sensors. The charge model: for every charger, the battery capacity is  $P$ , the travelling speed is  $v$ , and energy consumed by travelling one unit distance is  $c$ ,  $\eta_1$  is the charging efficiency between a charger and a sensor node,  $\eta_2$  is the charging efficiency among chargers. In order to unify with general problem, we also call the charger station as base station.

**Reasonable of Our CMCTW Problem.** Before explaining the difference between our CMCTW problem and the general problem, we give three practical conditions of WSNs and two assumptions.

Three practical conditions: 1) If we need schedule mobile chargers to charge the WSNs frequently in a short time, it may be more reasonable to partition all sensor nodes into several groups and place a charge station at the center of each group to charge these sensors in the group periodic by wireless charge. 2) Each node consumes energy for sensing, data reception and transmission. These processes frequently occur conducted by events they monitoring which obeying different parameter value of exponential distribute in difference environments. 3) The charge station gets the residual energy information of all sensor nodes before schedule mobile chargers to charge the WSNs.

Two assumptions: 1) Short Duration(SD): the duration of a charging is negligible compared to the traveling time of mobile chargers. 2) Long Cycle(LC): the recharging cycle of a sensor node is longer than a charging round.(it means any two consecutive charging rounds have no intersections.) It is easy to find that these assumptions can explained by practical condition.

### 3. Problem Formulation

The general Mobile Charge problem was described as

WSNs( $X, Y, B, T$ ) model and Charge model  $(P, c, v, \eta_1, \eta_2)$  [12], we also describe our problem as WSNs model and charge model. In this section, we will illustrate the difference between our specific mobile charge problem and general mobile charge problem one by one. Then, explain the reason why we choose specific like this.

We followed the methods of Zhao [1] based on the paper [2], which extends the original mobile charging problem from onedimensional application scenarios (such as highways) to twodimensional scenarios (similar to farmland, forest). This article, on the basis of [1], found practical application constraints, namely the time limit of charging time, and added time window constraints to re-formulate the problem. On the other hand, the quantitative analysis between the number of charger and the width of the two dimensions is added to further study the problem.

#### 3.1. Our Mobile Charge Model

We first give our specific mobile charge model: WSNs( $V, E, B, R, TW, D$ ) and  $(M, P, c, v, \eta_1)$ . Where  $V = \{v_0, v_1, v_2, \dots, v_n\}$  is sensor nodes set and  $v_i$  represents the  $i$ -th sensor node,  $v_0$  denotes the base station especially.  $E = \{d_{ij} = (v_i, v_j) | v_i, v_j \in V\}$  is the weight set where  $d_{ij}$  represent the Euclidean Distance from  $v_i$  to  $v_j$ . Node and weight sets  $V, E$  tells us the distribution of wireless sensor networks, and we can know the sensors residual energy and energy consuming rate by  $B = \{b_1, b_2, \dots, b_n\}$  and  $R = \{r_1, r_2, \dots, r_n\}$  (every sensor nodes has different residual energy and energy consuming, we will explain the reason following). The following charge model is similar to the general mobile charge problem:  $M$  mobile chargers are homogeneous, and for every charger, the battery capacity is  $P$ , the travelling speed is  $v$ , and energy consumed by travelling one unit distance is  $c$ ,  $\eta_1$  is the charging efficiency between a charger and a sensor node. Both travelling and wireless charging share the same battery on a mobile charger.

It's easy to find that TW and D are the additional conditions in our model. Where TW is the time windows set for every nodes, and D is the radius of the wireless sensor networks (we assume the WSNs as a circular area and sensor nodes distribute in it). Table I gives the list of all notations.

1. **Different Energy consuming rate:** The work [24] aims to scheduling the work sequencing of sensor nodes to maximize the time of sustain the WSNs work due to the condition 2). So, the energy consuming rate of each sensor node in WSNs is difference in different environments according to the work addressed in [24].
2. **Time Windows:** With the assumption of Long Cycle(LC), the mobile charge problem focus on a random time during a recharge cycle when some sensor nodes exhausted. Due to the different energy consuming rate, the residual energy of all sensor nodes must be not common when charge station decide to schedule mobile chargers. With an intuitive observation that, charging a sensor node one time with  $2q$  energy must be more effectiveness than charge a sensor node  $q$  energy twice in a periodic. So we assume to charge a sensor node while it's residual energy less than blow. And this is also the result of the assumption of LC, when you charge a sensor node with a residual energy larger than blow, the energy may keep the sensor node working without recharge in several charging rounds and still with residual energy larger than blow. All of this, we give the time windows  $TW = \{[l_1, u_1], [l_2, u_2], \dots, [l_n, u_n]\}$  of all sensor nodes which means a mobile charger must charge sensor node  $v_i$  during time interval  $[l_i, u_i]$ .
3. **Radius of WSNs:** Under a determined charge model  $(M, P, v, c, \eta_1)$ . It is obviously that mobile chargers can't serve the sensors distribute at bordering of the WSNs while the radius of WSNs is too large. There exists quantitative relationship between the radius of WSNs and the paraments of charge model  $(M, P, v, c, \eta_1)$ .

### 3.2. Objective Function and Constraints

With the above analysis, we can determine which WSNs  $(V, E, B, R, TW, D)$  model we can solve while the charge model  $(M, P, v, c, \eta_1)$  is fixed. We will deeply discuss the detail of the quantitative relationship between WSNs model and charge model in next section. Now, we give the mathematical description of our specific mobile charge problem include objective function and constraints.

Firstly, we use a new metric residual energy rate, differ to the energy usage effectiveness (EUE), as the new objective function. The metric of **Residual Energy Rate (RER)** is defined as:

$$RER = \frac{\sum_{i=1}^n F b_i}{nb} \quad (1)$$

TABLE I: List of Notations

WSN model	Definition
$V$	Set of sensor nodes
$E$	Weight set of sensor nodes in $V$
$b_i$	Current battery energy status of sensor $i$
$r_i$	Approximate energy rate of sensor $i$
$[l_i, u_i]$	Time windows of sensor $i$ , lower boundary $l_i$ and upper boundary $u_i$
$D$	Radius of the network
Charge model	Definition
$M$	Set of mobile charger
$P$	Battery capacity of mobile charger
$v$	Travelling speed of mobile charger
$c$	Energy consume rate of mobile charger
$\eta_1$	Charging efficiency between a charger and a sensor node
$\eta_2$	Charging efficiency between two chargers

Where  $Fb_i$  denotes the energy residual of sensor node  $v_i$  while mobile chargers finish this charging round.  $n$  denotes the number of sensor nodes in the WSNs.  $b$  denotes the battery capacity of sensor node (every sensor node has the same battery capacity).

The objective function of general mobile charge problem is energy usage effectiveness (EUE) which describes the effectiveness usage of the energy from base station. This objective function is to maximum the energy of charge to sensor nodes and minimum the energy loss during the charge process. In this paper, we aim to use the total residual rate of energy at the end of one charge round to evaluate a mobile charge process. It is obvious that a mobile charge processing is bad when the processing ended with low level of total energy residual level. It is possible that an example may get the high EUE value but low RER value. this may because of the random condition choose of WSNs( $V, E, B, R, TW, D$ ) not according to the charge model ( $M, P, v, c, \eta_1$ ). So, the RER may not only the evaluation metric of schedule, but also the evaluation matric of the reasonable of condition choose.

So far, we can give our brief mathematic model of this CM-CTW problem according to above analysis result as equation (2).

We can describe our specific mobile charge problem as a vehicle routing problem with time windows. Where the base station schedule  $M$  mobile chargers simultaneously, charge all sensor nodes which energy residual are lower than  $b_{low}$ (we call these nodes as desire nodes) and all chargers back to base station when all sensor nodes' residual energy is larger than  $b_{low}$ (formulate as the fifth equation in constraints). Fourth equation in constraints is the time which the fleet mobile chargers serve the WSNs. Just because of the constraint of serve time, some sensor nodes may consuming their energy lower than  $b_{low}$  during the serve time. So we call these sensor nodes as hidden nodes. Desire nodes and hidden nodes combine the serving sensor nodes  $\{\sigma_1, \sigma_2, \dots, \sigma_k, \dots, \sigma_{n1}\}$  which mobile chargers need to serve. Left sensor nodes  $\{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_s, \dots, \varepsilon_{n2}\}$  are unnecessary to serve by mobile chargers, and  $n1+n2 = n$ . So we can find the first and second equation in constraints denote the eventual residual energy of serving sensor nodes and rest sensor nodes respectively. Third equation in constraints means every sensor node from desire nodes set only charged by one mobile charger one time. Last two equations in constraints represent every desire sensor node must be serve during time windows and give the upper bound of mobile charger number. The correct expression of  $m$  will be given in next section.

$$\begin{aligned}
 Max \quad RER &= \frac{\sum_{i=1}^n Fb_i}{nb} & (2) \\
 s.t. \quad Fb_{\sigma_k} &= b - (T - t_{i,\sigma_k})r_{\sigma_k} & \sigma_k \in \left\{ \bigcup_{i=1}^M R_i \right\} \\
 Fb_{\varepsilon_s} &= b_{\varepsilon_s} - T \cdot r_{\varepsilon_s} & \varepsilon_s \in \left\{ V - \bigcup_{i=1}^M R_i \right\} \\
 R_i \cap R_j &= \{v_0\} & i \neq j \in \{1, \dots, M\} \\
 T &= \max \left\{ \frac{d(R_i)}{v} \right\} & i = \{1, \dots, M\} \\
 Fb_i &\geq b_{low} & i = \{1, \dots, n\} \\
 l_{\sigma_k} &\leq t_{i,\sigma_k} \leq u_{\sigma_k} \\
 M &\leq m
 \end{aligned}$$

#### 4. Quantitative Relationship Among Parameters

With the analysis of prior section, our CMCTW problem can be simplified into a schedule problem. The base station schedule a fleet of mobile chargers to charging these desire sensor nodes one time in the WSNs and back to base station, so that to keep the energy level of all sensor nodes larger than a energy level  $b_{up}$ .

In this section, we will analyse the relationship between WSNs model and charge model. Then, we give the quantitative relationship of variable between charge model and WSNs model in detail.

Energy of consuming rate can be got through the related work [24], and time windows of sensor nodes TW are determined by energy residual B. An obvious insight analysis is that the size of WSNs cannot be very large with a fixed level of residual energy of sensor node and charge model. Moreover, a determined charge model must consider the level of residual energy and WSNs size in order to charging the WSNs success.

We emphasize the Long Cycle assumption again before our quantitative analysis. Our assumption is not a hard constraint, it just keep a fact that any two consecutive charging rounds have no intersections. And the physical meaning of our next quantitative analysis can be reasonable explanation by this assumption.

#### 4.1. Several Known Variables Expression

Firstly, we can give the lower bound and upper bound time windows of sensor node  $v_i$  as follows:

$$u_i = \frac{b_i}{r_i}, l_i = \begin{cases} \frac{b_i - b_{low}}{r_i}, & \text{if } b_i > b_{up} \\ t_c, & \text{if } b_i \leq b_{up} \end{cases} \quad (3)$$

Where  $b_{up}$  is the upper bound of the sensor battery energy (as shown in Fig.1(b)), we use it to select desire node and determine lower bound of time windows.

An intuition analysis that a sensor node's rest work time must larger than the time of a charger moving from base station to this node. So we can give the distribution of all sensor nodes residual energy  $b_i \sim U[\frac{d_{1i}r_i}{v}, b]$  when we schedule a fleet mobile chargers to charge the WSNs. They obey uniform distribution with parameters  $\frac{d_{1i}r_i}{v}, b$ , where  $b$  is the battery capacity of sensor node.

With the analysis above, we give the following equation to determine an appropriate  $b_{low}$ .

$$\begin{aligned} q_1 &\leq E \leq q_2 \\ E &= \sum_{i=1}^n h(b_i < b_{up}) \cdot p(b_i < b_{up}) \\ p(b_i < b_{up}) &= \int_{\frac{d_{1i}r_i}{v}}^{b_{up}} \frac{v}{vb - d_{1i}r_i} dx, \frac{d_{1i}r_i}{v} < b_{up} \\ p(b_i < b_{up}) &= 0, \frac{d_{1i}r_i}{v} \geq b_{up} \end{aligned} \quad (4)$$

Where  $h(b_i < b_{up})$  denotes the event of  $b_i < b_{up}$ , and  $h(b_i < b_{up})$  constant equal to 1 ( $i = 1, 2, \dots, n$ ). The first constraint is mathematical expectation of events  $b_i < b_{up}$ , it represents the average number of desire nodes. The parameters  $q_1$  and  $q_2$  are the region of expect number of desire nodes that we give prior. It is easy to understand why we give these two parameters. If we made  $E$  too small, it may unnecessary to schedule charger and waite more desire nodes is more effectiveness. On the contrary, it may complex the schedule problem. The second constraint, where the integrand function is density function for the energy residual of sensor nodes, third constraint describes the relationship between  $p(b_i < b_{up})$  and  $b_{up}$ , so that we can obtain the appropriate  $b_{up}$  with above equations.

#### 4.2. Number of Mobile Charger

Then, we give the low bound of mobile charger number. A clear understand of relationship between mobile charger number  $M$  and mobile charger routing is that the better schedule of mobile charger the less number of mobile charger necessary. So, it may impossible to give an accuracy mobile charger number before scheduling the routing. An extreme situation is that mobile charger number  $M$  equal to sensor node number  $n$ . Another challenge to given the mobile charger number is the constraint of time windows. Several sensor nodes can't be served by one mobile charger in time because of their location distribution. So, we give an approximation of lower bound of  $M$  with these two aspects.

**For Energy Consumption:** First, we estimate the lower bound of M through the perspective of energy consumption.

The energy consumed in replenishing a WSN contains two parts: the energy charging to sensors, the energy consumed by chargers' travelling. Although it is hard to exactly get the length of travelling distance, we can give the mean of all edges  $d = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n d_{ij}$  to instead the average length of one edge in travelling when we know the mobile charger number.

$$M \cdot P \geq c(n_1 + M) \cdot d + \frac{1}{\eta_1} \sum_{k=1}^{n_1} (b - b_{\sigma_k})$$

$$M \geq M^{ec} = \frac{n_1 d c + \frac{1}{\eta_1} \sum_{k=1}^{n_1} (b - b_{\sigma_k})}{P - d \cdot c} \quad (5)$$

This just a approximation lower bound Mec and larger than the real lower bound from the perspective of energy consumption.

**For Time Windows:** Second, we estimate the lower bound of M through the time windows. Before give the mathematical description for lower bound of mobile charger number M, we define a theory called synchronous set. Synchronous set  $V_i$  represents a set of sensor nodes, that they have high probability can't be charged in their time windows by one mobile charger after it charging sensor  $v_i$ . We call this probability as synchronous probability and give the mathematical definition as follow:

$$p_{im} = 1 - \frac{\| [l_i + \frac{d_{im}}{v}, u_i + \frac{d_{im}}{v}] \cap [l_m, u_m] \|}{\| [l_m, u_m] \|} \quad (6)$$

$[l_i + \frac{d_{im}}{v}, u_i + \frac{d_{im}}{v}]$  is mobile charger star move from time windows  $[l_i, u_i]$  of sensor node  $v_i$  to sensor node  $v_m$ . So, the length of its intersection with time windows of sensor  $v_m$  can describe the asynchronous degree between  $v_i$  and  $v_m$ . A synchronous probability can give by 1 minus asynchronous probability which described by asynchronous degree divide length of time windows length to  $v_m$ .

So that we can give another time windows lower bound of mobile charger number Mtw as follows:

$$V'_i = \{v_m | p_{im} \geq p', m \neq i\}$$

$$M^{tw} = \max\{\|V'_i\| + 1 | i = 1, 2, \dots, n\} \quad (7)$$

$$M \geq M^{tw}$$

Where Mtw is the maximum size of all synchronous sets, then we can sustain all sensors operating. So far, we can give our approximate lower bound of mobile charger number  $\max\{Mec, Mtw\}$ .

### 4.3. Size of WSNs: D

With the residual energy region  $[\frac{d_{1i} r_i}{v}, b]$ , we can get a large upper bound of D.

$$\frac{d_{1i} r_i}{v} < b$$

$$d_{1i} < \frac{b \cdot v}{r_i} \quad (8)$$

Through this simply relative between lower and upper bound of residual energy, we obtain the upper bound of  $d_{1i}$ . And D approximate equal to maximum  $d_{1i}$  as follows:

$$D = \max\{\frac{b \cdot v}{r_i} | i = 1, 2, \dots, n\} \quad (9)$$

But, it may be an extreme situation when increase the mobile charger number. So we given the through energy and mobile charger number as follows:

$$D \leq \frac{1}{2} \cdot \frac{M \cdot P - \frac{1}{\eta_1} \sum_{k=1}^{n_1} (b - b_{\sigma_k})}{M \cdot c} \quad (10)$$

This is just a smaller upper bound of WSNs size D. While the charge model is fixed and residual energy distribution suitable, a D satisfied equation (10) can be solved. The upper bound of D must be larger, and we will give a more correct D in our further work.

Fig. 3 shows the motivational situation of equation(10). We assume the left picture of Fig. 3 is a solution of this problem, and obtain the right picture through tensile these routings. We use the average value of D1,D2 and D3 to represent the upper bound of D. This is the physical meaning of equation (10).

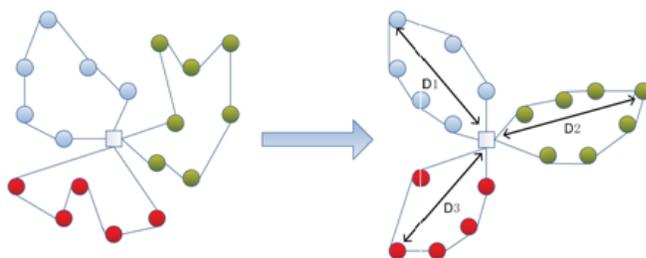


Fig. 3. Explain of the upper bound of D

## 5. Our Scheduling Algorithm

In this section, the scheduling algorithm will be given to solve our CMCTW problem when WSNs model and charge model are determined with the above analysis.

### 5.1. Adding Virtual Node

Our CMCTW problem can be translated into a Vehicle Routing Problem with Time Windows (SVRPTW). We perform this transform to improve the quality of solution and do local optimization after building solution, because the single routing problem is easier than multiple routing problem. So, we translate our multiple routing problem into single routing problem through adding virtual charger station. The Fig. 4 intuitive explain the processing of transform.

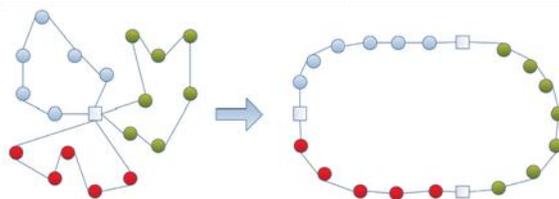


Fig. 4. Translate of the virtual nodes

The left figure in Fig. 4 is the initial routing, and the right figure is the routing after transform. The addition two charger stations (square) are virtual node, it means a mobile charger arrive charger station in practical when it arrive virtual node in figure (the distance among virtual node and other sensor nodes are equal to the distance among charger station and other sensor nodes). With this transform, our CMCTW problem the left figure in Fig. 4 is translated into Hamilton cycle that each node in graph must be visited if and only if one time. This transform is also prepared for the local optimization in section V-C.

### 5.2. Main Rules of Our Algorithm

To build a feasible routing which satisfied time windows and energy constraint, we design some building rules as follows.

First, we build a candidate set to provide mobile charger selecting a suitable sensor node to charge according to its current residual energy and time.  $P_c$  and  $T_c$  is the current residual energy and current time of a charger. A sensor  $v_i$  belong to a charger's candidate set  $V_{candi}$  when the charger's residual energy can provide the charger move from its current node to  $v_i$  charging it and back to base station. Another constraint is that the charger arrive to  $v_i$  must during the  $v_i$  time windows. So we give the following candidate set.

$$V_{candi} = \{v_i | P_c > \frac{b_i}{\eta_1} + c(d_{ci} + d_{i1}), l_i < T_c + \frac{d_{ci}}{v} < u_i\} \quad (11)$$

$$V_{candi} \cap V_{visited} = \{\emptyset\}$$

Where equation (11) is candidate set while mobile charger has already charging a set of sensor nodes  $V_{visited}$  and arrive at  $vc$ . And the candidate set  $V_{candi}$  must no intersection with visited node set  $V_{visited}$ .

When the candidate set  $V_{candi} = \{\emptyset\}$ , the mobile charger must return back to base station. We will give the pseudo-code of our algorithm in Algorithm 1.

With the transform in section V-A, the algorithm can be regarded as a greedy algorithm to solving a constraint TSP problem.  $n + M$  is the number of total node and with  $M - 1$  virtual nodes. Line 2-5 obtains the candidate set in step  $i$  according to history routing and current residual energy and current time. Adding the nearest node in routing and refresh

$T_c = T_c + d_{ri-1,ri}/v, E_c = E_c - d_{ri-1,ri} \cdot c - (B - b_i) - r_i \cdot T_c$  if the candidate set is not empty, otherwise, refresh the current  $T_c = 0, E_c = P$ . After  $n + M$  steps (line 1-15), the solution has built and do some improvement (line 16-17).

### 5.3. Local Optimization

After we obtain the solution through our algorithm, we use a local optimization to get a better solution.

We use an efficient and simply algorithm 2-opt [25] to optimize the solution locally. The optimize processing as shown in Fig. 5 when the multiple routing problem has been transformed into single routing problem through using virtual base station.

We can see the longer routing  $i \rightarrow i + 1 \rightarrow \dots \rightarrow j \rightarrow j + 1$  has optimized to shorter routing  $i \rightarrow j \rightarrow \dots \rightarrow i + 1 \rightarrow j + 1$  with the routing direction change from  $i + 1 \rightarrow \dots \rightarrow j$  to  $j \rightarrow \dots \rightarrow i + 1$ . And an important issue of the 2-opt is that we need to keep the tour feasible while we change the **Algorithm 1** Schedule Algorithm for VRPTW routing direction from  $i + 1 \rightarrow \dots \rightarrow j$  to  $j \rightarrow \dots \rightarrow i + 1$ . We design to performance this 2-opt when direction change not break the energy and time windows constraints. The process above is based on triangle inequality.

**Require:** WSNs model  $(V, E, B, R, TW, D)$  and Charge Model  $(M, P, v, c, \eta)$ ;  
**Ensure:** State variables current time  $T_c$  and current energy  $E_c$ , current routing sequence  $R$ .

- 1: **for**  $i = 1; i \leq n + M; i++$  **do**
- 2: calculate unvisited sensor nodes set  $V_{unvisited}$  with current sequence  $R = \{r_1, r_2, \dots, r_{i-1}\}$ ;
- 3: calculate feasible sensor nodes set  $V_{tw}$  about time windows with current time  $T_c$ ;
- 4: calculate feasible sensor nodes set  $V_{ec}$  about energy constraint with current energy  $E_c$ ;
- 5: obtain the candidate feasible sensor nodes set  $V_{candi} = V_{unvisited} \cap V_{tw} \cap V_{ec}$ ;
- 6: **if**  $V_{candi} \neq \emptyset$  **then**
- 7:  $r_i = \text{argmin}_{j \in V_{candi}} \{d_{ri-1, j}\}$ ;
- 8: Add  $r_i$  to  $R$ ;
- 9: Refresh parameters about  $T_c, E_c$ ;
- 10: **else**
- 11:  $r_i = \text{BaseStation}$ ;
- 12: Add  $r_i$  to  $R$ ;
- 13: Refresh parameters about  $T_c = 0, E_c = P$ ;
- 14: **end if**
- 15: **end for**
- 16: Using 2-opt to optimize the solution  $R$ .
- 17: Using collaboration to optimize the solution  $R$ .
- 18: Calculate the objective function **RER**.

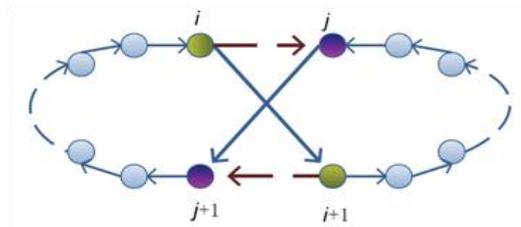


Fig. 5. Processing of local optimization

#### 5.4. Collaboration between mobile charger

Finally, we use collaboration among mobile chargers to optimize the solution.

After we built a feasible routing and using 2-opt to improve the tour, we consider whether we should improve the solution further more. Such as the left picture in Fig. 6, the routing  $i \rightarrow i + 1, \rightarrow, \dots$  cross with routing  $j \rightarrow j + 1, \rightarrow, \dots$ . We can improve it like the right picture, but the feasibility of the change must be kept. This improvement based on the wireless energy transfer and collaboration of chargers.

#### 5.5. The Complexity of Algorithm

In Algorithm 1, line 2-4 are the processes to calculate node sets. All of them are finishing in linear time  $O(n)$ . In line 6-14, except line 7 is linear time complexity  $O(n)$ , the rest processes are constant complexity  $O(1)$ . And the process in line 16-17 are at most  $O(n^2)$ . Thus, the total time complexity is  $O(n^2)$ .

### 6. Performance Evaluation

Three different simulations would be done in this section, and we will give the analysis in detail.

## 6.1. Simulation Setup and Quantitative Check

We assume that sensor nodes are powered by a 1.5V

2000mAh Alkaline rechargeable battery, then the battery capacity (b) is  $1.5V \times 2A \times 3600sec = 10.8KJ$ ; the battery capacity of a mobile charger (P) is 2000KJ, moving speed of charger (v) is 1m/s; the charger's moving cost (c) is 10J/m. With the analysis of section 4, we first give the size of WSNs  $D = \frac{b \cdot v}{1.5V \times 2A/2}$ , this is based on the equation (9).

We assume that wireless sensor nodes are uniformly deployed over a  $[-D \times D]$  2-dimensional square area. The base station BS is located at the center of the square, and the coordinates of BS is (0,0). By default, the number of sensor nodes (N) is set 200. After determining the location of a sensor node  $v_i$ , we can know the energy consuming rate  $r_i$ .

We assume, with a given  $q_1$  and  $q_2$ , we can obtain blow by equations (4). So that, we can give the lower and upper bound of sensor node  $v_i$   $[l_i, u_i]$  by equation (3). Finally, the lower bound of mobile charger number can be given by equation (5)-(7). Another way of simulation setup: if we have already known the mobile charger number M, we can give the size of WSNs D by equation (10) and the rest parameters can be give as similar as above. With the setup above, we can keep our words that the WSNs model can be charged by charge model feasibly.

The link: [26] is a public Baidu cloud link where stores our experiment data. The data format is '.tsp' and can be opened by text. Each data file include the specific value of all parameters include (M,P,c,v, $\eta_1, \eta_2, q_1, q_2, D$ ) and coordinate set of sensor node V, distance matrix E, energy consuming rate set R. Then, time windows set TW can be conducted by V and R.

We compare our ScheduleAlgorithm with  $H\eta$ ClusterCharging( $\beta$ ) algorithm and their improvement ScheduleAlgorithm + 2opt,  $H\eta$ ClusterCharging( $\beta$ ) + 2opt, 0 + 2opt0 (in section V-C) means we adding a local search algorithm called '2opt' [25] in this algorithm again. We compare these four algorithms through residual energy rate, mobile charger number and feasible node rate respectively.

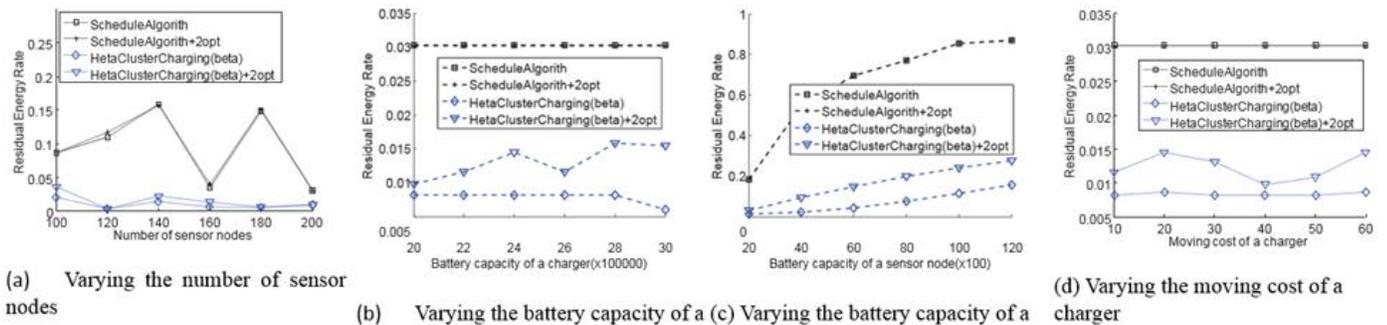


Fig. 7. Performance comparisons for RER

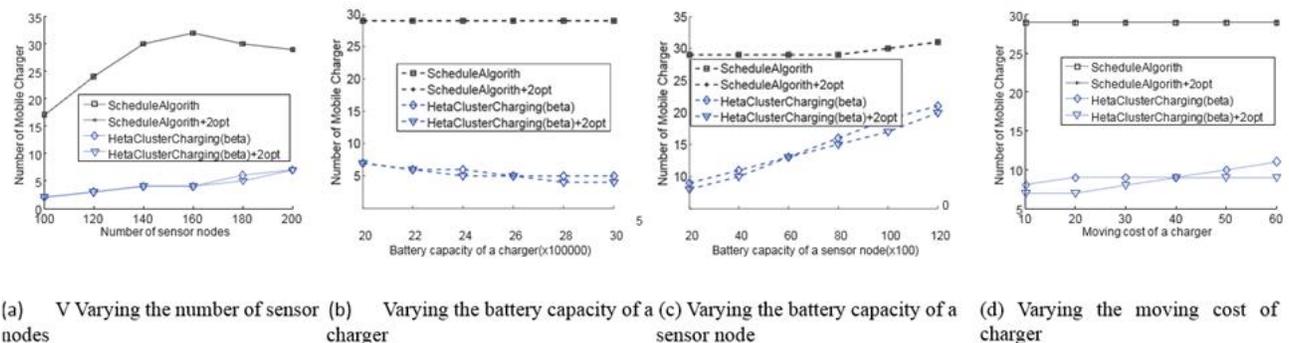


Fig. 8

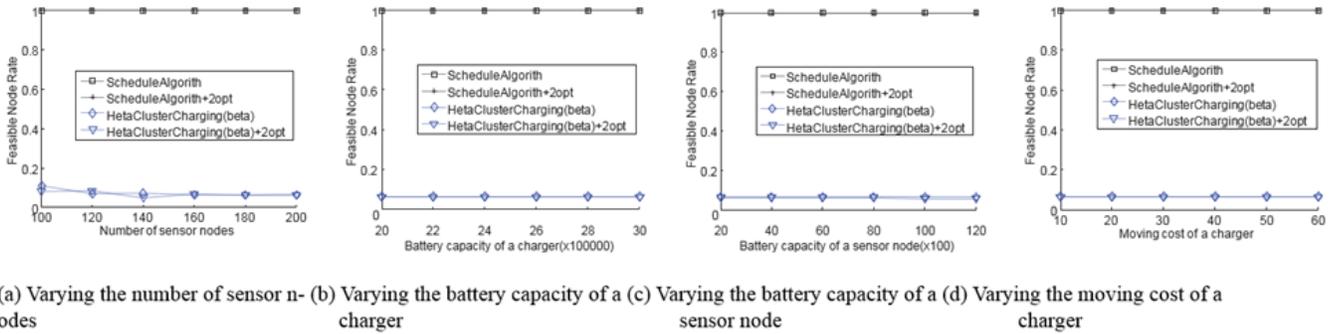


Fig. 9. Performance comparisons for feasible nodes rate

Where heuristic algorithm  $H\eta$ ClusterCharging( $\beta$ ) proposed in [12], it can obtain the art-of-state solution in 1-dimension collaborative mobile charge problem.

## 6.2. Compare Results in Residual Energy Rate

Fig. 7 expresses the RER value with these four algorithm while the sensor number, battery capacity of a charger, battery capacity of a sensor node, moving cost of a charger variety. An obvious look is that, algorithm with '2opt' is better than the algorithm simply and improvement of '2opt' on ScheduleAlgorithm is limited. It is because our algorithm ScheduleAlgorithm has already improved by local optimization and collaboration. With the result of Fig. 7, our ScheduleAlgorithm all better than the  $H\eta$ ClusterCharging( $\beta$ ) algorithm on RER.

In Fig. 7(a), when the number of sensor nodes increases, mobile chargers may cost more time to finish a charging round, this means more energy would be cost by sensor nodes. According to this, all of the four algorithms perform worse when the number of sensor nodes decrease. A few wave will be shown because of the specificity of every example. In Fig. 7(c), as the battery capacity of a sensor node increase, the RER of all four algorithms also increase. This is reasonable, since charger can deliver more energy to a sensor node in one cycle. In Fig. 7(b) and Fig. 7(d), where the RER is no change with the increase of charger battery capacity and moving cost. It may be because of the no relevant among RER, moving cost and charger battery capacity. As the increase of battery capacity of charger, it may decrease the charger number we used to charge, it may still cannot decrease the maximum travelling time of mobile chargers. So, there is no change for RER with the increase of charger battery capacity.

## 6.3. Compare Results in Mobile Charger Number And Feasible Nodes Rate

Firstly, we would give the definition of feasible node rate. Feasible node rate represents the number of sensor node which charged by charger during their time windows ratio to the total sensor nodes number.

In Fig 8, we can see the charger number used in our algorithm ScheduleAlgorithm is larger than it in  $H\eta$ ClusterCharging( $\beta$ ). It may be a huge disadvantage for our algorithm with a first sight, however, it would be overlooked to compare with the hundred percent of feasible node rate in Fig. 9. That's because we consider the constraint of time windows as a hard constraint. The purpose we exhibit Fig. 8 and Fig. 9 is that the extra mobile chargers can be made up by the the high feasible node rate.

In Fig.9,our ScheduleAlgorithm and ScheduleAlgorithm+2opt are coincidence. No matter how optimization we adding in the algorithm, the time windows must be satisfied. And the  $H\eta$ ClusterCharging( $\beta$ ) and  $H\eta$ ClusterCharging( $\beta$ ) + 2opt in Fig. 9(b), Fig. 9(c) and Fig. 9(d) are coincidence and stable, that represents the feasible node rate would not influenced by battery capacity of charger, battery capacity of sensor node and moving cost. The slight wave of  $H\eta$ ClusterCharging( $\beta$ ) and  $H\eta$ ClusterCharging( $\beta$ ) + 2opt in Fig. 9(a) are reasonable, and may be caused by the specific of the example.

From Fig. 8, we find that it is easy to understand the vary of charger number in  $H\eta$ ClusterCharging( $\beta$ ) and  $H\eta$ ClusterCharging( $\beta$ ) + 2opt. Charger number decrease when battery capacity of charger increased. And charger number increase when moving cost, battery capacity of sensor and number of sensor nodes increased.

Now, we would analysis the charger number in our ScheduleAlgorithm change with these parameters. And how to decrease the number of mobile charger that used in the problem is our further work. In Fig. 8(b), the charger number has no change with the increase of battery capacity of charger. That can be explained by equation (6)-(7), the charger number may restraint by the time windows that means many sensor nodes need charge, and their can not be charged by one charger due to their location. So, the charger number in this problem with time window may be no relevant with charger battery capacity. The phenomenon in Fig. 8(d) is similar. The increase of our ScheduleAlgorithm and ScheduleAlgorithm+2opt in Fig. 8(b) can be explained by equation (5). When  $b$  increases, and  $M_{ec}$  increased larger than  $M_{tw}$ .  $M = \max\{M_{ec}, M_{tw}\}$ , so the number of mobile charger increase.

## 7. Conclusion

In this paper, we investigate the mobile charging scheduling problem in WSNs. By introducing energy charging bound, we translate the problem into classic problem vehicle routing problem with time windows (VRPTW). To analyze the problem deeply, we give the quantitative relationship among problem model and solution model. We add  $M$  virtual base stations to translate the multiple routing problem into single routing problem and using local optimization and collaboration to optimize the solution. And the results of simulation validate the advantages of our algorithm.

In our further work, we would like improve our algorithm to decrease the charger number and increase the residual energy rate, try to using some intelligent algorithm. In problem, we want to consider several WSNs model charging simultaneously, and collaboration can be done among different WSNs models.

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