ISBN 978-93-84422-80-6
10th International Conference on Law, Business, Marketing and Education (LBME-17)
Pattaya (Thailand) Dec. 27-28, 2017

A Comparison of Envelopment Models for Selecting Performance Measures

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Abstract**:** *Data envelopment analysis (DEA) is a methodology to evaluate decision making units (DMUs) with similar tasks in a production system that produces multiple outputs with consuming multiple inputs. In the evaluation process of DEA for each DMU a relative efficiency score is find by solving a linear programming problem. When the number of DMUs is less than the number of performance measures, DEA models evaluate a large number of DMUs as efficient and this situation is a challenge. In order to deal with this challenge, one solution method is ignoring some performance measures and selecting problem of these measures under the different assumptions makes a new question. This paper is concerned with the comparison of two individual and aggregate selecting approaches under constant and variable returns-to-scale assumptions. A real dataset including 20 banks in Iran is employed to explain the performance of these approaches in selection process of selective measures.*

Keywords: *Data envelopment analysis; selective measures; envelopment form; selection process; baking industry.*

1. Introduction

Optimization techniques can be used to estimate the performance efficiency of firms if we know the functional forms for the relationships among various performance measures. Data Envelopment Analysis (DEA) is an optimization-based methodology which introduced by Charnes, et al. [1] has been proven an effective tool in evaluating the homogeneous firms and finding a relative efficiency for each firm. DEA generates a single relative efficiency score, while considering multiple inputs and outputs simultaneously. To this end, for evaluating n homogeneous decision making units (DMUs), which has m inputs and s outputs, as performance measures, DEA defines an efficient frontier and uses mathematical programming implicitly to estimate the tradeoffs inherent in the efficient frontier. Along with the speedy advances in mathematical programming and operations research, DEA method has been rapidly developed. In the course of this development, some critical challenges have occurred [2]. One of the most important of these challenges occurs when the number of DMUs is low in comparison with the number of performance measures; in this situation, most of the DMUs are assessed efficient and hence the obtained results are questionable. On the other hand, clumsily ignoring of some measures from considerations can extremely change the real position of DMUs. Therefore, selection of input and output items is crucial for successful application of DEA [5]. A rough rule of thumb in the DEA model is to choose the number of DMUs equal to or greater than $\max\{3(m+s), m \times s\}$ [5]. In some real-world problems, the number of performance measures and the number of DMUs do not satisfy the rule of thumb. In such situations, selecting a number of appropriate measures is an important issue. A variety of researchers attempted to tackle this issue: Toloo et al. [3] proposed two individual DMU and aggregate models to develop the idea of selective measures, under the constant returns-to-scale (CRS) assumption.Toloo and Tichý [6] improved Toloo et al.'s [3] models with the aim of developing a pair of alternative approaches for selecting performance measures under variable returns-to-scale (VRS) assumption. Keshavarz and Toloo [7] proposed a single stage approach for selecting inputs/outputs in DEA, based on the VRS assumption and the common set of weights methodology. Toloo and Allahyar [8] extended an envelopment form of selecting the model of Toloo et al. [3].

The current paper attempts to compar two individual and aggregate of selecting models under CRS and VRS assumption, from the envelopment form point of view. The content of this paper is organized in the following way: Section 2 provides a review of the standard envelopment form models of DEA satisfying the CRS and VRS assumptions. In Sections 3, two individual-based selecting models under CRS and VRS assumptions are reviewed. A pair of aggregate models is formulated in Section 4 to select the adequate performance measures for CRS and VRS technologies. In order to show the applicability and compare the proposed models, in Section 5, a real data set of bank industry is used. Conclusions and future researchers are provided in the last section.

2. Standard DEA Models

The first two basic DEA models are originated by Charnes, Cooper and Rhodes in 1978 and Banker, Charnes and Cooper in 1984 and hence these models are known as the CCR and BCC models. The former model is formulated for CRS assumption meanwhile the latter model deals with VRS assumption. Assume that there are n DMUs and each DMU_j $(j = 1,\dots,n)$ uses m semi-positive inputs $\boldsymbol{x}_j = (x_{1j},\dots,x_{mj})$to produce s semi-positive outputs $\boldsymbol{y}_j = (y_{1j},\dots,y_{sj})$. The following pair of models measures the CCR- and BCC-efficiency score of unit under evaluation, i.e. DMU_o, respectively:

$$
\begin{array}{ll}
\max \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^x + \sum_{r=1}^{s} s_r^y\right) & \\
\text{s.t.} & \\
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta x_{io} & \forall i \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^y = y_{ro} & \forall r \\
\lambda_j \geq 0 \ , s_i^x \geq 0 \ , s_r^y \geq 0 & \forall j, \forall i, \forall r
\end{array}
\tag{1}
$$

$$
\begin{array}{ll}
\max \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^x + \sum_{r=1}^{s} s_r^y\right) & \\
\text{s.t.} & \\
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta x_{io} & \forall i \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^y = y_{ro} & \forall r \\
\sum_{j=1}^{n} \lambda_j = 1 & \\
\lambda_j \geq 0 \ , s_i^x \geq 0 \ , s_r^y \geq 0 & \forall j, \forall i, \forall r
\end{array}
\tag{2}
$$

where λ_j is the intensity variable, s_i^x and s_r^y are the i^{th} input and r^{th} output slacks, respectively, $\varepsilon > 0$ is the non-Archimedean infinitesimal (for more details about the role of non-Archimedean infinitesimal in DEA models we refer the readers to [9]). These models differ from the convexity constraint $\sum_{j=1}^{n} \lambda_j = 1$. These envelopment form of CCR and BCC models seeks a (virtual) unit $(\sum_{j=i}^{n} \lambda_j^* \mathbf{x}_j \, , \sum_{j=1}^{n} \lambda_j^* \mathbf{y}_j)$ where its minimum output level is $\mathbf{y}_o$ in all components, from the constraint $\sum_{j=1}^{n} \lambda_j^* \mathbf{y}_{rj} \ \geq y_{ro}; r = 1,\dots,m,$ while reducing the input vector $\mathbf{x}_o$ proportionally to a value as small as possible, from the constraint $\sum_{j=1}^{n} \lambda_j^* \, \mathbf{x}_{ij} \leq \theta^* x_{io};\ i = 1,\dots,m.$ If the objective value of models (1) and (2) is equal to one, (or equivalently $\theta_o^* = 1$, $\forall i\ s_i^{x^*} = 0$ and $\forall r\ s_r^{y^*} = 0$), then DMU_o is CCR- and BCC-efficient. Otherwise $(\sum_{j=i}^{n} \lambda_j^* \mathbf{x}_j \, , \sum_{j=1}^{n} \lambda_j^* \mathbf{y}_j)$ outperforms $(\theta^* \mathbf{x}_o, \mathbf{y}_o)$ which means the unit under evaluation is inefficient under either CRS or VRS assumptions. As inspection makes clear, CCR-efficiency score of each unit is less than or equal to its BCC-efficiency score [5]. However, these models might be useless if the number of performance measures and DMUs does not meet the rule of thumb. In the next two sections, we review a pair of individual- and aggregate-based approach in the envelopment forms under CRs and VRS assumption with the aim of dealing with selective measures.

3. Individual-Based Approach

Assume that $n < max\{3(m+s), m \times s\}$ and a considerable number of the CCR- and BCC-efficiency scores obtained by the aforementioned models is one. There some solutions to deal with this issue (i) adding some weight restrictions (ii) increasing the number of DMUs or (ii) decreasing the number of performance measures. For more details about the assurance region method we refer the readers to [10]. Form a practical point of view, in most often cases, it is not too easy to add some DMUs. Toloo and Tichý [6] developed an approach to opt some performance measures such that the rule of thumb is met. Let s_1 and s_2 denote subsets of outputs corresponding to fixed-output (which are selected by the decision maker) and selective-output measures (which are selected by a selecting model), respectively. Similarly, assume that m_1 and m_2 are the parallel subsets of inputs. Following T Toloo and Tichý [6] the following pair of envelopment form of individual-based CRS and VRS selecting models can be used to select a subset of performance measures:

$$
\begin{array}{ll}
\max = \theta - \varepsilon\left(\sum_{i\in m_1} s_i^x + \sum_{r\in s_1} s_r^y\right) - \varepsilon\left(\sum_{i\in m_2} t_i^x + \sum_{r\in s_2} t_r^y\right) & \\
\text{s.t.} & \\
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta x_{io} & \forall i \in m_1 \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^y = y_{ro} & \forall r \in s_1 \\
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta x_{io} + M(1-d_i^x) & \forall i \in m_2 \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^y = y_{ro} - M\left(1-d_r^y\right) & \forall r \in s_2 \\
\sum_{i\in m_2} d_i^x = p & \\
\sum_{r\in s_2} d_r^y = q & \\
0 \le t_i^x \le M d_i^x & \forall i \in m_2 \\
s_i^x - M(1-d_i^x) \le t_i^x \le s_i^x & \forall i \in m_2 \\
0 \le t_r^y \le M d_r^y & \forall r \in s_2 \\
s_r^y - M\left(1-d_r^y\right) \le t_r^y \le s_r^y & \forall r \in s_2 \\
d_i^x, d_r^y \in \{0,1\}, t_i^x, t_r^y \ge 0 & \forall i \in m_2, \forall r \in s_2 \\
s_i^x, s_r^y, \lambda_j \ge 0 & \forall i, \forall r, \forall j
\end{array} \tag{3}
$$

$$
\begin{array}{ll}
\max = \theta - \varepsilon\left(\sum_{i\in m_1} s_i^x + \sum_{r\in s_1} s_r^y\right) - \varepsilon\left(\sum_{i\in m_2} t_i^x + \sum_{r\in s_2} t_r^y\right) & \\
\text{s.t.} & \\
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta x_{io} & \forall i \in m_1 \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^y = y_{ro} & \forall r \in s_1 \\
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta x_{io} + M(1-d_i^x) & \forall i \in m_2 \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^y = y_{ro} - M\left(1-d_r^y\right) & \forall r \in s_2 \\
\sum_{i\in m_2} d_i^x = p & \\
\sum_{r\in s_2} d_r^y = q & \\
\sum_{j=1}^{n} \lambda_j = 1 & \\
0 \le t_i^x \le M d_i^x & \forall i \in m_2 \\
s_i^x - M(1-d_i^x) \le t_i^x \le s_i^x & \forall i \in m_2 \\
0 \le t_r^y \le M d_r^y & \forall r \in s_2 \\
s_r^y - M\left(1-d_r^y\right) \le t_r^y \le s_r^y & \forall r \in s_2 \\
d_i^x, d_r^y \in \{0,1\}, t_i^x, t_r^y \ge 0 & \forall i \in m_2, \forall r \in s_2 \\
s_i^x, s_r^y, \lambda_j \ge 0 & \forall i, \forall r, \forall j
\end{array} \tag{4}
$$

Where M is a large positive number, b_i^x and b_r^y are indicator variables correspond to selective input $i \in m_2$ and selective output $r \in s_2$, respectively. In these models we have:

$$
b_i^x = \begin{cases} 1, & i^{th} \text{ input is selected} \\ 0, & \text{Otherwise} \end{cases}, \quad b_r^y = \begin{cases} 1, & r^{th} \text{ output is selected} \\ 0, & \text{Otherwise} \end{cases}
$$

In addition, parameters $p, q \in \mathbb{N}$ are natural numbers and represent the number of selected inputs and outputs, respectively. Following theorem shows the condition that models (3) and (4) comply the rule of thumb.

Theorem 1. The presented models (3) and (4) will meet the rule of thumb if $p + q \leq \min\left\{\left[\frac{n}{3}\right], 2\sqrt{n}\right\} - (|m_1| + |s_1|)$.

Proof. Toloo and Tichý [6] ■

Models (3) and (4) accommodate performance measures with a pessimistic standpoint under CRS and VRS assumptions. In other words, this approach aims at enhancing the discriminating power of selecting models with measuring the minimum possible efficiency score of unit under evaluation. However, when the discriminating power of individual-based approach is not sharp enough the aggregate-based approach can be utilized. In contract with the individual-based approach which should be solved for each unit, the aggregate-based is solved for an aggregate DMU involving aggregate inputs $(\sum_{j=1}^{n} x_{1j}, \dots, \sum_{j=1}^{n} x_{mj})$ and aggregate outputs $(\sum_{j=1}^{n} y_{1j}, \dots, \sum_{j=1}^{n} y_{sj})$. Next section presents a pair of aggregate-based envelopment approach with CRS and VRS assumptions.

4. Aggregate-Based Approach

Aggregate-based approach deals with the overall performance of the collection of DMUs for accommodate selective measures. In this approach, the performance efficiency of the aggregate outputs to aggregate inputs with the outlook on selective measures. The following MIP models measure the aggregate efficiency under CRS and VRS assumptions, respectively.

$$
\begin{array}{ll}
\max = \theta - \varepsilon\left(\sum_{i\in m_1} s_i^x + \sum_{r\in s_1} s_r^y\right) - \varepsilon\left(\sum_{i\in m_2} t_i^x + \sum_{r\in s_2} t_r^y\right) & \\
\text{s.t.} & \\
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta\left(\sum_{j=1}^{n} x_{ij}\right) & \forall i \in m_1 \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^y = \left(\sum_{j=1}^{n} y_{rj}\right) & \forall r \in s_1 \\
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta\left(\sum_{j=1}^{n} x_{ij}\right) + M(1 - d_i^x) & \forall i \in m_2 \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^y = \left(\sum_{j=1}^{n} y_{rj}\right) - M\left(1 - d_r^y\right) & \forall r \in s_2 \\
\sum_{i\in m_2} d_i^x = p & \\
\sum_{r\in s_2} d_r^y = q & \\
0 \leq t_i^x \leq M d_i^x & \forall i \in m_2 \\
s_i^x - M(1 - d_i^x) \leq t_i^x \leq s_i^x & \forall i \in m_2 \\
0 \leq t_r^y \leq M d_r^y & \forall r \in s_2 \\
s_r^y - M\left(1 - d_r^y\right) \leq t_r^y \leq s_r^y & \forall r \in s_2 \\
d_i^x, d_r^y \in \{0,1\}, t_i^x, t_r^y \geq 0 & \forall i \in m_2, \forall r \in s_2 \\
s_i^x, s_r^y, \lambda_j \geq 0 & \forall i, \forall r, \forall j
\end{array}
\tag{5}
$$

$$
\begin{aligned}
&\max = \theta - \varepsilon\left(\sum_{i\in m_1} s_i^x + \sum_{r\in s_1} s_r^y\right) - \varepsilon\left(\sum_{i\in m_2} t_i^x + \sum_{r\in s_2} t_r^y\right) && \\
&\text{s.t.} && \\
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta\left(\sum_{j=1}^{n} x_{ij}\right) && \forall i \in m_1 \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^y = \left(\sum_{j=1}^{n} y_{rj}\right) && \forall r \in s_1 \\
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta\left(\sum_{j=1}^{n} x_{ij}\right) + M(1-d_i^x) && \forall i \in m_2 \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^y = \left(\sum_{j=1}^{n} y_{rj}\right) - M\left(1-d_r^y\right) && \forall r \in s_2 \\
&\sum_{i\in m_2} d_i^x = p && \\
&\sum_{r\in s_2} d_r^y = q && \\
&\sum_{j=1}^{n} \lambda_j = 1 && \\
&0 \le t_i^x \le M d_i^x && \forall i \in m_2 \\
&s_i^x - M(1-d_i^x) \le t_i^x \le s_i^x && \forall i \in m_2 \\
&0 \le t_r^y \le M d_r^y && \forall r \in s_2 \\
&s_r^y - M\left(1-d_r^y\right) \le t_r^y \le s_r^y && \forall r \in s_2 \\
&d_i^x, d_r^y \in \{0,1\}, t_i^x, t_r^y \ge 0 && \forall i \in m_2, \forall r \in s_2 \\
&s_i^x, s_r^y \ge 0 && \forall i, \forall r \\
&\lambda_j \ge 0 && \forall j
\end{aligned}
\tag{6}
$$

where M is a large positive number, d_i^x and d_r^y are indicator variables correspond to selective input $i \in m_2$ and selective output $r \in s_2$, respectively. In these models, if $d_i^x = 1$ for $i \in m_2$ then (i) we have $s_i^x \le t_i^x \le s_i^x$ from the constraint $s_i^x - M(1-d_i^x) \le t_i^x \le s_i^x$ which leads to $t_i^x = s_i^x$ (ii) from the constraint $\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta\left(\sum_{j=1}^{n} x_{ij}\right) + M(1-d_i^x)$ we obtain $\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^x = \theta\left(\sum_{j=1}^{n} x_{ij}\right)$. As a result, the i^{th} input is selected. On the other hand, if $d_i^x = 0$, then (i) the constraint $\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^y = \left(\sum_{j=1}^{n} y_{rj}\right) - M$ is redundant and (ii) $t_i^x = 0$ for the constraint $0 \le t_i^x \le M d_i^x$. In this case, we can conclude that the i^{th} should not be selected. In the same way, we can interpret the role of indicator variable d_r^y. Hence, we have the following relations:

$$
d_i^x = \begin{cases} 1, & i^{th} \text{ input is selected} \\ 0, & \text{Otherwise} \end{cases}, \quad d_r^y = \begin{cases} 1, & r^{th} \text{ output is selected} \\ 0, & \text{Otherwise} \end{cases}
$$

Next section provides a real dataset of banking industry in Iran to illustrate how the aforementioned approaches select performance measures.

5. Case Study

We use a real dataset involving 20 branches of the largest private bank in Iran to compare the selected performance measures with individual- and aggregate- based approaches under CRS and VRS assumptions. TABLE demonstrates the data set including six inputs; Employees (x_1), Number of accounts (x_2), Assets (x_3), Space (x_4), Costs (x_5), and Expenses (x_6); and six outputs; Number of transactions (y_1), Deposits (y_2), Loans (y_3), Check card (y_4), Credit card (y_5), and OTP[1] (y_{6s}). For a sake of simplicity, we assume that all inputs and outputs are selective measures $(s_1 = m_1 = \emptyset)$ and all inputs and outputs are selective measures.

The last two columns in TABLE shows that 80% of banks (i.e. 16 out of 20) are CCR-efficient and 95% of banks (i.e. 19 out of 20) are BCC-efficient. These results are questionable because the large number of performance measures exists in comparison with the number of DMUs $(20 = n < 36 = \max\{3(m+s), m \times s\}$. To obtain acceptable result, we have to select the number of performance measure such that the rule of thumb is satisfied, i.e. n excess $3(m + s)$. For this purpose, we first solve the individual-based selecting models (3) and (4), by assuming $p = 3$ and $q = 3$, as a managerial suggestion which satisfies the condition of Theorem 1. TABLE shows the optimal values of binary variables b_i^x $(i = 1, \ldots, 5)$ and b_r^y $(r = 1, \ldots, 4)$; these variables characterize selected measures. As can be seen, in both CRS and VRS individual models x_1, x_3, x_4, y_2, y_3, and y_6 which have maximum frequency have been selected measures. The first two columns of

[1] One-Time Password.

TABLE show the efficiency scores of all banks in the presence of these selected measures. As it can be seen, the percentage of CCR- and BCC-efficient banks is reduced to 15% and 30%, respectively.

TABLE I: Bank Data and their CCR and BCC Efficiency Scores.

Banks	Inputs						Outputs						CCR-efficiency	BCC-efficiency
	(x_1)	(x_2)	(x_3)	(x_4)	(x_5)	(x_6)	(y_1)	(y_2)	(y_3)	(y_4)	(y_5)	(y_6)		
1	11	1250	1753	97	10020	3137	5214	72149	57537	5105	4839	25	1.0000	1.0000
2	17	5019	2604	150	11440	4406	5343	89781	51114	8646	8364	24	0.9689	1.0000
3	7	3217	1155	61	8427	2180	5145	42654	52485	2797	2697	5	1.0000	1.0000
4	12	1061	1899	105	11816	6477	3249	97812	67298	3373	3096	68	1.0000	1.0000
5	14	5219	2215	123	12426	3325	6706	77031	43487	8993	8787	58	1.0000	1.0000
6	14	1389	2357	123	9907	3757	6259	75923	41442	7604	7371	40	1.0000	1.0000
7	9	7166	1370	79	10365	2714	3652	47763	43262	3608	3497	9	0.6846	0.7799
8	5	1475	829	44	5283	2887	3913	45732	14237	3795	3500	32	1.0000	1.0000
9	6	1800	985	52	11061	2852	3566	55222	41062	3299	3182	15	0.9350	1.0000
10	6	1689	1023	52	5856	2606	4559	53323	37418	1858	1746	8	1.0000	1.0000
11	8	1780	1311	70	8745	4442	4441	69734	57883	3030	2882	23	1.0000	1.0000
12	9	2669	1536	79	7326	1989	5031	49153	47139	4811	4578	31	1.0000	1.0000
13	8	7175	1367	70	8326	3727	5053	92365	55543	6840	6588	45	1.0000	1.0000
14	7	2120	1193	61	6525	3473	4762	64235	22347	5382	5188	22	1.0000	1.0000
15	9	30618	1359	79	8158	3824	6876	89104	45717	7628	7292	105	1.0000	1.0000
16	7	1464	1111	61	11135	1524	4307	42012	73925	3187	2984	22	1.0000	1.0000
17	7	8924	1182	68	6920	3573	5331	69360	27246	3743	3524	24	1.0000	1.0000
18	7	2388	1069	61	5864	2523	4004	51438	26531	4360	4140	17	0.9946	1.0000
19	6	4714	992	52	5039	2398	2342	39948	20223	2688	2574	36	1.0000	1.0000
20	7	1866	1180	62	8378	3165	4238	154284	43928	4182	4008	18	1.0000	1.0000

In order to do an adaptive comparison, we solve the aggregate selecting models (5) and (6). The results point out that model (5) selects x_1, x_3, x_4, y_2, y_4 , and y_5 measures meanwhile model (6) identifies selects x_1, x_2, x_4, y_1, y_2, and y_3 measures. It should be noted that, both CRS and VRS aggregate approaches opt x_1, x_4, and y_2 measures. The last two columns of TABLE exhibit the efficiency scores of all banks obtained under CRS and VRS technologies in the presence of the selected measures. As we expected, the percentage CCR- and BCC-efficient DMUs via the aggregated-based approaches is changed to 15% and 65% which illustrate that the discriminating power of aggregate-based approach is not better than the individual-based approaches.

TABLE II: Results of Solving Models (3) and (4) for Data Set

Model (3)

Banks	b_i^x						b_r^y					
	(x_1)	(x_2)	(x_3)	(x_4)	(x_5)	(x_6)	(y_1)	(y_2)	(y_3)	(y_4)	(y_5)	(y_6)
1	1	0	1	1	0	0	0	1	1	0	0	1
2	1	1	0	1	0	0	0	1	1	0	0	1
3	0	1	1	0	0	1	0	0	0	1	1	1
4	1	0	0	1	0	1	1	0	0	1	1	0
5	1	0	1	1	0	0	0	1	1	0	0	1
6	1	0	1	1	0	0	0	1	1	0	0	1
7	1	0	0	1	1	0	0	1	0	1	0	1
8	0	0	1	0	1	1	0	1	1	0	0	1
9	0	1	0	0	1	1	0	0	0	1	1	1
10	0	1	1	0	0	1	0	0	0	1	1	1
11	1	0	1	0	0	1	0	0	0	1	1	1
12	1	0	1	1	0	0	0	1	1	0	0	1
13	0	1	1	0	0	1	1	1	1	0	0	0
14	1	0	1	0	0	1	0	1	1	0	0	1
15	1	1	0	1	0	0	0	1	1	0	1	0
16	1	0	0	1	1	0	0	1	0	0	1	1
17	0	1	0	1	0	1	0	1	1	0	0	1
18	1	0	0	1	0	1	0	1	1	0	0	1
19	1	0	1	1	0	0	0	0	1	1	1	0
20	0	0	1	0	1	1	0	0	0	1	1	1
Sum	13	7	12	12	5	11	2	13	12	8	9	16

Model (4)

Banks	b_i^x						b_r^y					
	(x_1)	(x_2)	(x_3)	(x_4)	(x_5)	(x_6)	(y_1)	(y_2)	(y_3)	(y_4)	(y_5)	(y_6)
1	1	0	1	1	0	0	0	1	1	0	0	1
2	1	1	0	1	0	0	0	1	1	0	0	1
3	1	1	1	0	0	0	0	1	0	1	0	1
4	1	0	0	1	0	1	1	0	0	1	1	0
5	1	0	1	1	0	0	0	1	1	0	0	1
6	1	0	1	1	0	0	0	1	1	0	0	1
7	1	1	0	1	0	0	1	0	0	1	0	1
8	1	0	1	0	0	1	0	0	1	1	0	1
9	0	1	0	0	1	1	1	0	1	0	0	1
10	1	1	1	0	0	0	0	0	0	1	1	1
11	0	0	1	1	0	1	0	0	0	1	1	1
12	1	0	1	1	0	0	0	1	1	0	0	1
13	1	0	1	0	0	1	1	1	0	0	0	1
14	1	1	1	0	0	0	0	1	1	0	0	1
15	0	0	1	1	0	1	1	1	0	1	0	0
16	1	0	0	1	1	0	1	1	0	0	0	1
17	0	1	0	1	0	1	0	0	1	1	0	1
18	1	1	0	1	0	0	1	1	0	0	0	1
19	1	1	0	1	0	0	1	1	1	0	0	0
20	0	1	1	0	0	1	1	0	1	0	0	1
Sum	15	10	12	13	2	8	9	12	11	8	3	17

TABLE III: Efficiency Scores Obtained by Individual And Aggregate Approaches				
Banks	Individual-based approach		Aggregate-based approach	
	CCR-efficiency	BCC-efficiency	CCR-efficiency	BCC-efficiency
1	0.6008	0.6267	0.5625	1.0000
2	0.4004	0.4220	0.6062	0.5962
3	0.7518	0.8999	0.4913	1.0000
4	0.7822	1.0000	0.4433	1.0000
5	0.4897	0.5228	0.7701	1.0000
6	0.4301	0.4765	0.6407	1.0000
7	0.5426	0.7106	0.4908	0.6675
8	0.6944	1.0000	0.8897	1.0000
9	0.7834	1.0000	0.6981	1.0000
10	0.7165	0.9787	0.4723	1.0000
11	0.8073	0.8411	0.5295	0.8911
12	0.6166	0.7006	0.6253	0.8503
13	0.9482	0.9497	1.0000	1.0000
14	0.5413	0.7849	0.9037	0.9756
15	1.0000	1.0000	1.0000	1.0000
16	1.0000	1.0000	0.5455	1.0000
17	0.6106	0.8083	0.6853	1.0000
18	0.5436	0.8414	0.7390	0.7879
19	0.6091	0.9109	0.5472	0.8789
20	1.0000	1.0000	1.0000	1.0000

6. Conclusion and Future Researchers

The paper deals with the problem of selecting measures in DEA models to do a comparison between the individual- and aggregate-based envelopment form models under CRS and VRS assumptions. Our results noticed that the number of efficient DMUs is significantly decreased via both individual- and aggregate-based approaches under CRS technology, also in the VRS technology individual-based model decreases the number of efficient DMUs, but performance of aggregate-based approach was not defensible. It is also shown that different selective measures might be selected by considering different models. A case study is utilized to illustrate the provided comparison. An interesting future research topic is formulating other selecting models with the other thachnologies or assumptions.

7. Acknowledgements

The research was supported by the Czech Science Foundation through project No. 17-23495S.

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