Numerical Study of an Open Cracked Rotor

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Abstract: *In this paper, we study the effect of an open transverse crack on the vibratory behavior of a rotor using the classical version of MEF, the finite element Euler-Bernoulli beam is used for the discretization of this rotor. The equation of motion is obtained by the application of the Lagrange equation on the kinetic and deformation energies of the different components of the rotor taking into account the variation of the stiffness of the shaft due to the crack, A program which allows us to plot the Campbell diagram to determine the natural and critical frequencies of a cracked and uncracked rotor is developed in MATLAB. After the validation of the results found by our program with results of a simulation with ANSYS Workbench which we have done, and also with experimental results. We study the influence of the position and the depth of the crack on the natural and critical frequencies, this study helps us to detect cracks in the rotor's shaft.*

Keywords: *Rotor, Open crack, FEM, Campbell diagram,*

1. Introduction

Rotors have extensive applications in many industrial machines, they are widely used in turbines, compressors, pumps, motors ... etc. Continuous loads on the rotors can lead to unpredictable damage and failure such as cracks. This damage leads to dangerous, destructive and catastrophic scenarios.

Over the past three decades, many researchers have worked and provided theoretical and experimental work to study the dynamics of cracked and uncracked rotors. Comments on cracked rotors can be found in [1-3]. The breathing and the open model of the transverse crack are considered to be the main theories to study the vibratory behaviour of cracked rotor systems.

The presence of cracks causes a local variation in the stiffness of the shaft of the rotor. The techniques used to formulate this variation, while using the FEM, are the flexibility matrix method and the method of the time-varying stiffness. The flexibility matrix method is the most common technique for formulating the stiffness matrix of a cracked element of the rotor [4-7], The time-varying stiffness method which uses the formulas of the moments of inertia of an asymmetric rod in space is translated by a reduction of the moments of inertia around the axes of rotation of the cracked element, this method was used in [8] and [9] to study the model of the open cracks and in [10-12] to study the breathing cracks.

Previous studies in the field of cracked rotors show that the response of the cracked rotor is slightly different from the uncracked rotor, the natural and critical frequencies of a cracked and uncracked rotor have a gap that can be used to identify the presence and depth and position of the crack.

In our work, we use the classical finite element method and the time-varying stiffness method to study the influence of an open transverse crack on the dynamic behavior of a rotor. This study makes it possible to determine the presence, as well as the depth and the position of the crack. A program that allows us to generate the Campbell diagram to identify the natural and critical frequencies of a cracked and uncracked rotor has been realized in MATLAB, after the validation of our results with a results that obtained by a simulation with ANSYS Workbench that we are done and with experimental results given by [9], we study the influence of the position and the depth crack on the natural and critical frequencies of a rotor.

2. Equation of Motion

The rotor consists of a shaft resting on bearings and comprising one or more disks. It can also have solicitations like unbalance or other external forces.

The equations of motion are obtained by the application of the Lagrange equation to the kinetic energies and deformation of the rotor components (shaft-disk-bearings):

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial \dot{q}_i}+\frac{\partial U}{\partial q_i}=F_i \tag{1}$$

2.1. Kinetic Energy of the Disk

The disk is considered rigid and therefore characterized by its kinetic energy only, the expression of this latter is:

$$T_d=T_d^{trans}+T_d^{rot}=\frac{1}{2}m_d\left(\dot{u}^2+\dot{w}^2\right)+\frac{1}{2}I_{dX}\left(\dot{\theta_X}^2+\dot{\theta_Z}^2\right)+\frac{1}{2}I_{dY}\left(\Omega^2+2\dot{\theta}_X\theta_Z\right) \tag{2}$$

Where

$$I_{dX}=I_{dZ}=\frac{m_d}{12}\left(3r^2+3R^2+h^2\right) \tag{3}$$

$$I_{dY}=\frac{m_d}{2}\left(r^2+R^2\right) \tag{4}$$

2.2. Kinetic and Deformation Energy of the Shaft

For an element of length L_e, the expression of the kinetic energy will be given by:

$$T_a=\frac{1}{2}\rho_a s\int_0^{Le}(\dot{u}^2+\dot{w}^2)\,dy+\rho_a\frac{I_a}{2}\int_0^{Le}(\dot{\theta_X}^2+\dot{\theta_Z}^2)\,dy+\rho_a I_a L_e\Omega^2+2\rho I_a\Omega\int_0^{Le}\dot{\theta}_X\,\theta_Z dy \tag{5}$$

The energy of deformation of the shaft is calculated by considering the case of a flexible beam in rotation without deformation at the shear force (Euler-Bernoulli beam), the expression of the energy of deformation of the shaft will be given by:

$$U_a=\frac{E}{2}\int_0^{Le} I_Z\left(\frac{\partial\theta_Z}{\partial y}\right)^2+I_X\left(\frac{\partial\theta_X}{\partial y}\right)^2+2I_{XZ}\frac{\partial\theta_Z}{\partial y}\frac{\partial\theta_X}{\partial y} \tag{6}$$

For a healthy element, the moments of inertia along the axes X and Z are equal to $I_X=I_Z=\pi R^2/4$ and $I_{XZ}=0$, but in the case of a cracked element where the section is not circular (Figure 1), the crack causes an asymmetry in the course of the rotation, according to ref [12] the moments of inertia along the x and z axes are given by:

$$\begin{cases} I_x=\frac{\pi R^4}{8}+\frac{R^4}{4}\left((1-\mu)(2\mu^2-4\mu+1)\gamma+\sin^{-1}(1-\mu)\right)\\ I_z=\frac{\pi R^4}{4}-\frac{R^4}{12}\left((1-\mu)(2\mu^2-4\mu-3)\gamma+3\sin^{-1}\gamma\right)\\ I_{xz}=-\frac{1}{2}\left((R\mu)^2(R\mu-2R)^2\right)\end{cases} \tag{7}$$

Where $\mu=h/R$ is the ratio between the depth of the crack and the radius of the shaft ($0\le\mu\le1$) and $\gamma=\sqrt{\mu(2-\mu)}$.

According to ref [13] the moments of inertia along the x and z axis (rotating reference) with respect to the moments of inertia along the axis X and Z (fixed reference) are:

$$\begin{cases} I_x=\frac{I_X+I_Z}{2}+\frac{I_X-I_Z}{2}\cos(2\Omega t)-I_{XZ}\sin(2\Omega t)\\ I_z=\frac{I_X+I_Z}{2}-\frac{I_X-I_Z}{2}\cos(2\Omega t)+I_{XZ}\sin(2\Omega t)\\ I_{xz}=\frac{I_X-I_Z}{2}\sin(2\Omega t)-I_{XZ}\cos(2\Omega t)\end{cases} \tag{8}$$

From equations (7) and (8) we deduce the relations of moments of inertia of a cracked section along the X and Z axes (fixed reference), which are given by:

$$\begin{cases} I_{XZ}=I_{xz}\cos^2(2\Omega t)-\frac{1}{2}\cos(2\Omega t)\sin(2\Omega t)\,(I_x-I_z)\\ I_Z=\frac{1}{2}\left(I_x+I_z+\frac{1}{\cos(2\Omega t)}(I_x-I_z+2I_{XZ}\sin(2\Omega t))\right)\\ I_X=I_x+I_z-I_Z\end{cases} \tag{9}$$

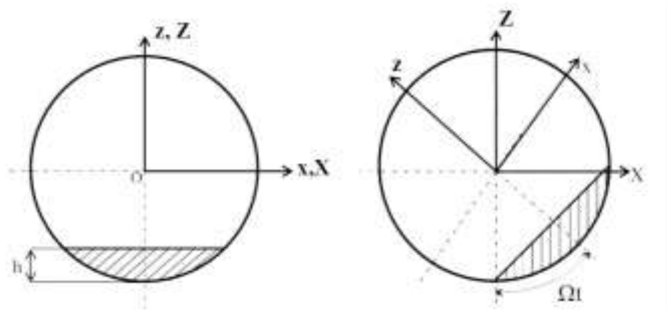

Fig. 1: Cross-section of a cracked element.

The moments of inertia in equation (9) are variable with respect to the angle of rotation Ωt, this means that the system is non-linear, whereas in the literature, the systems with open cracks are considered linear. In our work, we linearize the system of equation by taking the mean values of each moment.

2.3. Virtual Work of the Bearings

The virtual work of the forces due to the bearings acting on the shaft is:

$$\delta w = -k_{xx}u\delta u - k_{xz}w\delta u - k_{zz}w\delta w - k_{zx}u\delta w - c_{xx}\dot{u}\delta u - c_{xz}\dot{w}\delta u - c_{zz}\dot{w}\delta w - c_{zx}\dot{u}\delta w \qquad (10)$$

It can be put in the matrix form:

$$\begin{bmatrix} F_u \\ F_w \end{bmatrix} = -\begin{bmatrix} k_{xx} & k_{xz} \\ k_{zx} & k_{zz} \end{bmatrix}\begin{bmatrix} u \\ w \end{bmatrix} - \begin{bmatrix} c_{xx} & c_{xz} \\ c_{zx} & c_{zz} \end{bmatrix}\begin{bmatrix} \dot{u} \\ \dot{w} \end{bmatrix} \qquad (11)$$

Where F_u and F_w are the generalized force.

3. Matrix Formulation

After the application of the Lagrange equation on the kinetic and deformation energies of the rotor components, taking into account the variation in the shaft's stiffness due to the crack, we used classical version of the FEM and the time-varying method to determine the global matrices of the equation of motion [11]:

$$[M]\ddot{q} + ([C_p] + \Omega[G])\dot{q} + ([K_p] + [K])q = F \qquad (12)$$

Where:

[M] is the global mass matrix which comprises the mass matrix of the disk and the shaft.
[G] is the global gyroscopic matrix that includes the gyroscopic matrix of the disk and the shaft.
[K] is the global stiffness matrix of the shaft which comprises the stiffness matrix of the cracked element assembled with the matrices of other elements replacing the matrix of the healthy element by the matrix of cracked element.
$[C_p]$ and $[K_p]$ are the damping and stiffness matrices of the bearings.
q is the vector of generalized coordinates.
F is the vector of unbalance forces, in our work, this vector force is neglected.
Ω is the rotation speed in rd/s.

The natural and critical forward and backward frequencies are obtained from the eigen solution of the matrix given as:

$$S(\Omega) = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}G(\Omega) \end{bmatrix} \qquad (13)$$

4. Validation of Results

Figure 2 shows the finite element model of the rotor studied by [9], the rotor is discretized in 6 elements, Figure 3 shows the model studied in the ANSYS Workbench simulator, and the physical parameters of this rotor are presented in table 1.

Figures 4, 5, 6 and 7 present Campbell's diagrams of a cracked and uncracked rotor generated by our program and by ANSYS Workbench. From these diagrams we can determine the values of the natural and critical frequencies of different cases of position and depth of the crack.

Tables 2, 3 and 4 show the gap between the natural and critical frequency values found by our MATLAB program and the results found by ANSYS Workbench where the crack depth ratio equals to μ=0.46, and the crack is found in the second element where the distance between the left bearings and the crack is equal to 0.14 m.

Tables 4, 5 and 6 show the differences between the critical frequencies values of a cracked and uncracked rotor found by our MATLAB program and the results of the experimental [9].

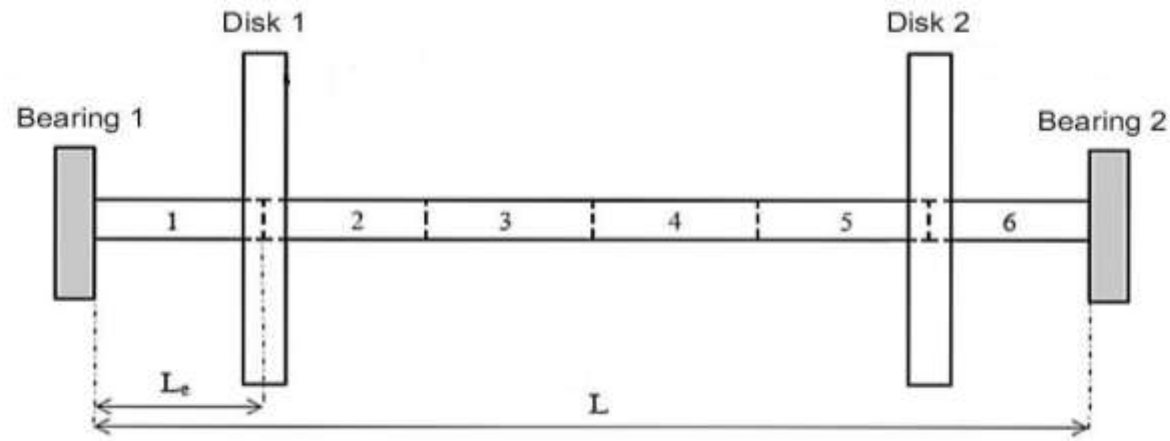

Fig. 2: Finite element model of the rotor studied.

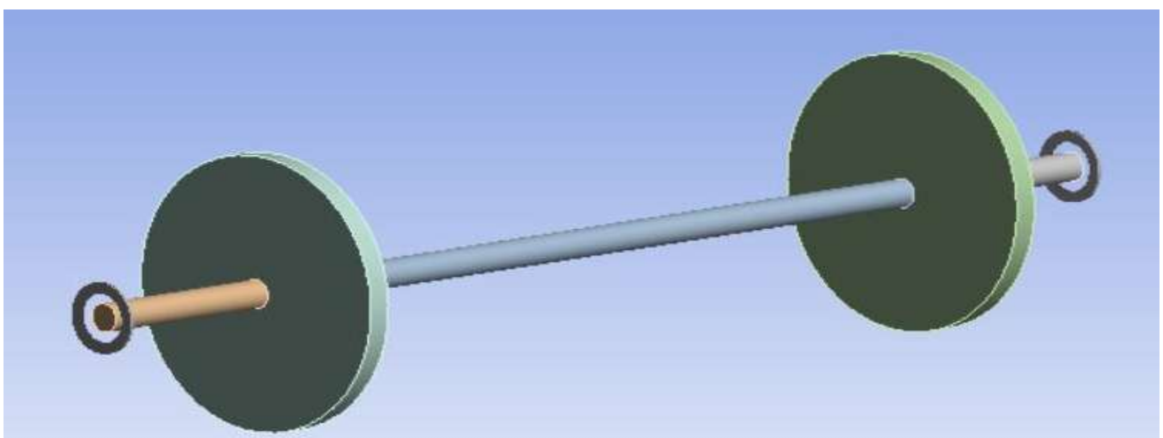

Fig. 3: Model of the rotor studied in ANSYS Workbench.

TABLE 1: Physical parameters of the rotor studied

Young's Module (E)	2.1e11 N/m2
Length of the shaft (L)	0.65 m
Density of the shaft (ρ_a)	7800 kg/m3(Steel)
Number of disks	2
Position of the disk 1	0.124 m (from the left bearings)
Position of the disk 2	0.6 m (from the right bearings)
Inner radius of the disks (R)	0.795e-3 m
External radius of the disks (r)	76.2e-3 m
Thickness of the disks (e)	11.72e-3 m
Density of the disk (ρ_d)	2700 kg/m3(Aluminum)
Stiffness of bearings 1 and 2 (k_{xx},k_{zz})	7e7 N/m
Damping of bearings 1 and 2 (c_{xx},c_{zz})	5e2 Ns/m

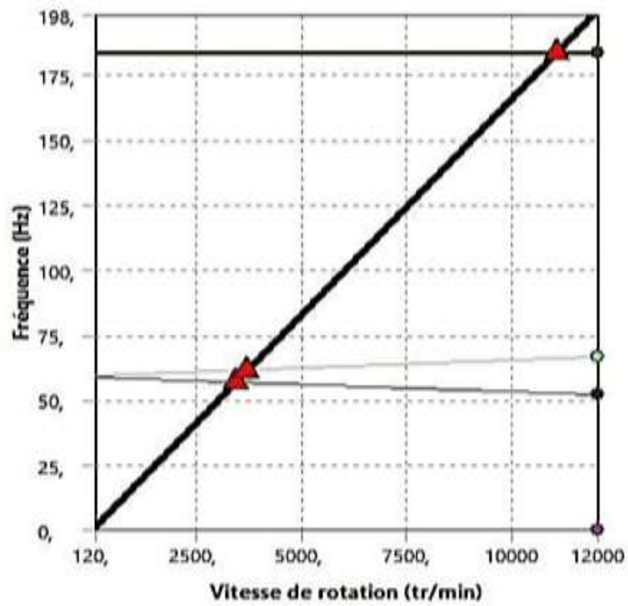

Fig. 4: Campbell diagram of uncracked rotor generated by ANSYS Workbench.

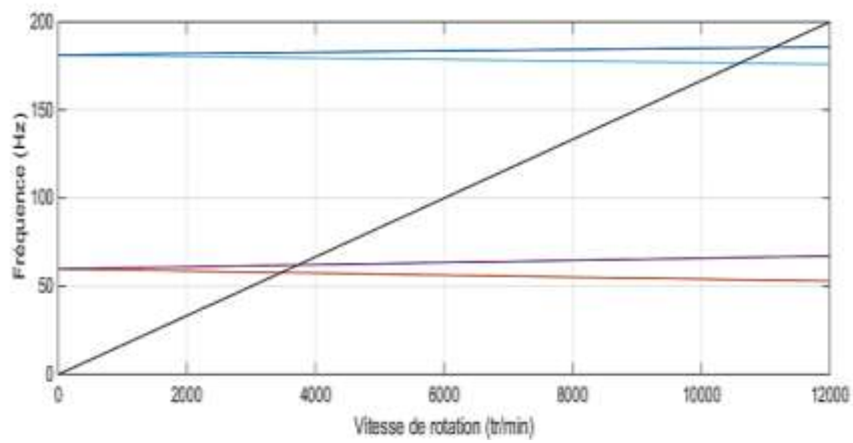

Fig. 5: Campbell diagram of uncracked rotor generated by our program of MATLAB.

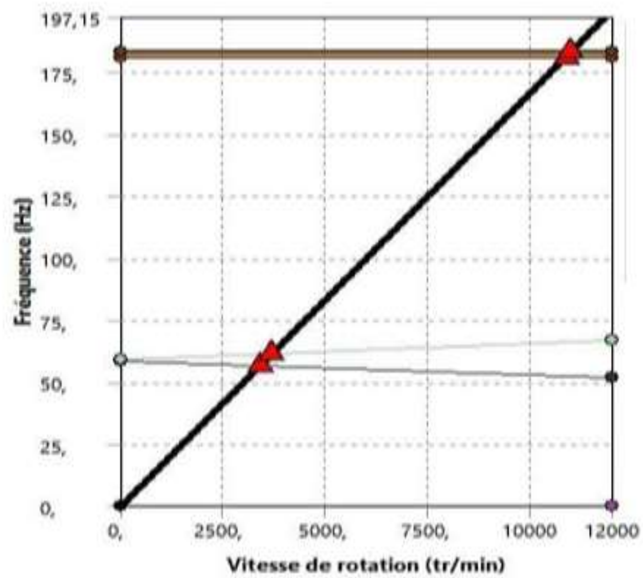

Fig. 6**:** Campbell diagram of cracked rotor generate by ANSYS Workbench where the crack is located in 0.14 m from the left bearing and μ=0.46.

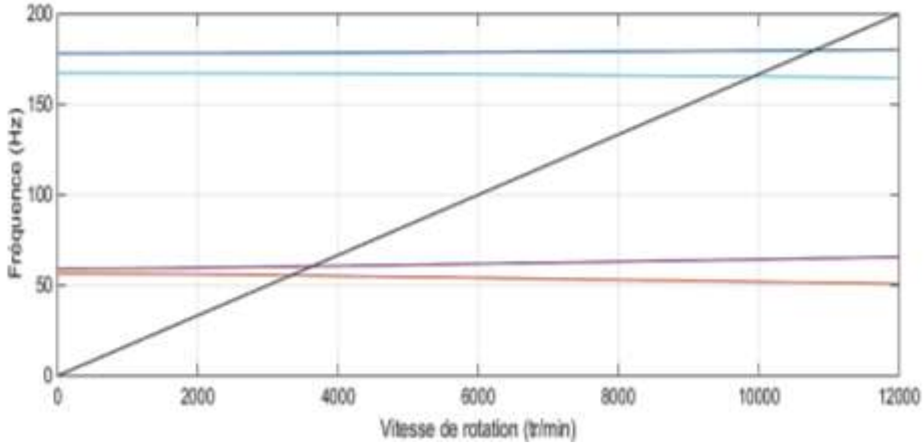

Fig. 7: Campbell diagram of uncracked rotor generate by our program of MATLAB where the crack is located in 0.14 m from the left bearing and μ=0.46.

TABLE II: Validation of the natural frequencies obtained by our program -MATLAB- with that obtained by ANSYS Workbench where the crack is located in 0.14 m from the left bearing and μ=0.46

Mode		Natural frequencies					
		Uncracked rotor			Cracked rotor		
		MATLAB	ANSYS Workbench	ε1 (%) (Mat/ANSYS)	MATLAB	ANSYS Workbench	ε2 (%) (Mat/ANSYS)
1	forward	59.88	58.83	1.78	55.36	58.6	5.5
	backward	59.88	59.08	1.35	59	59.08	0.13
2	forward	181.2	183.4	1.2	178.3	181.09	1.54
	backward	181.2	183.7	1.36	180.6	183.14	1.38

TABLE III: Validation of the critical frequencies obtained by our program -MATLAB- with that obtained by ANSYS Workbench where the crack located in the 2 element and μ=0.46

Mode		Critical frequencies					
		Uncracked rotor			Cracked rotor		
		MATLAB	ANSYS Workbench	ε3 (%) (Mat/ANSYS)	MATLAB	ANSYS Workbench	ε4 (%) (Mat/ANSYS)
1	forward	57.83	58	0.29	55.53	56.72	2.09
	backward	62.13	59.5	4.42	60.58	61.4	1.33
2	forward	176.7	183.4	3.65	165.4	181.08	8.65
	backward	185.5	183.7	0.98	179.9	183.15	1.77

TABLE IV: Validation of the critical frequencies obtained by our program -MATLAB- with that of experimental of uncracked rotor

Mode		Critical frequencies				
		Uncracked rotor				
		MATLAB	ANSYS Workbench	Experimental	ε5 (%) (Mat/Exp)	ε6 (%) (ANSYS/Exp)
1	forward	57.83	58	56.6	2.17	2.41
	backward	62.13	59.5	/	/	/

TABLE V: Validation of the critical frequencies obtained by our program -MATLAB- with that of experimental where the crack is located in 0.14 cm from the left bearing and μ=0.46

Mode		Cracked rotor				
		MATLAB	ANSYS Workbench	Experimental	ε7(%) (Mat/Exp)	ε8 (%) (ANSYS/Exp)
1	forward	55.53	56.72	/	/	/
	backward	60.58	61.4	60.7	0.19	1.15

TABLE VI: Validation of the critical frequencies obtained by our program -MATLAB- with that of experimental where the crack is located in 0.25 m from the left bearing and μ=0.46

Mode		Cracked rotor				
		MATLAB	ANSYS Workbench	Experimental	ε9 (%) (Mat/Exp)	ε10 (%) (ANSYS/Exp)
1	forward	54.52	56.49	/	/	/
	backward	60.03	61.3	60	0.05	2.17

5. Influences of the Position and Crack Depth on Natural and Critical Frequencies:

We study the influence of the depth and the position of the crack on the natural and critical frequencies of the rotor. We vary the depth of the crack, as well as the position of the crack.

Table 7, 8 show that the natural and critical frequencies of the backward and forward modes of the cracked rotor decrease with respect to that of an uncracked rotor, this decrease is slight in backward modes by that in forward modes.

The variation curve of natural and critical frequencies with respect to the position of the crack is similar to the shape of deformation mode, for example in the first mode, the frequencies decrease when we approach to the middle of the rotor's shaft.

Therefore, the results found show that the detection of the crack is difficult because we can find many results have the same values for different case of depth and position of the crack.

TABLE VII: Variation of the natural frequencies with respect to the crack depth and the position of the crack.

Cracked element	Mode		Ratio of the crack depth (μ)										
			0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	1	forward	59.88	59.83	59.71	59.54	59.31	59.01	58.61	58.1	57.46	56.67	55.75
		backward	59.88	59.85	59.82	59.79	59.76	59.74	59.72	59.7	59.67	59.62	59.55
	2	forward	181.2	180.7	179.6	178	175.8	173.1	169.7	165.5	160.7	155.4	149.8
		backward	181.2	180.9	180.6	180.3	180.1	179.9	179.7	179.5	179.1	178.7	178
2	1	forward	59.88	59.62	59.09	58.33	57.32	56.05	54.49	52.61	50.45	48.06	45.58
		backward	59.88	59.74	59.58	59.44	59.32	59.23	59.14	59.03	58.88	58.67	58.36
	2	forward	181.2	180	177.4	174.1	170.1	165.4	160.4	155.2	150.1	145.2	140.9
		backward	181.2	180.6	179.7	179.1	178.5	178.1	177.7	177.2	176.5	175.6	174.2
3	1	forward	59.88	59.5	58.75	57.68	56.32	54.56	52.66	50.38	47.84	45.16	42.48
		backward	59.88	59.68	59.44	59.24	59.08	58.95	58.82	58.67	58.46	58.16	57.73
	2	forward	181.2	181	180.5	179.8	178.9	177.8	176.6	175.1	173.6	171.9	170.1
		backward	181.2	181.1	180.9	180.8	180.7	180.6	180.5	180.4	180.3	180.1	179.8

TABLE VIII: Variation of the critical frequencies with respect to the crack depth and the position of the crack.

Cracked element	Mode		Ratio of the crack depth (μ)										
			0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	1	forward	57.83	57.83	57.73	57.67	57.53	57.38	57.18	56.87	56.45	55.87	55.12
		backward	62.13	62.07	61.97	61.87	61.75	61.55	61.33	61.08	60.85	60.55	60.28
	2	forward	176.7	176.3	175.7	174.8	173.5	171.6	168.8	165.2	160.3	155.4	150.3
		backward	185.5	184.9	184.1	183	182	181	180.3	179.4	178.8	178.7	177.9
2	1	forward	57.83	57.63	57.28	56.78	56.08	55.12	53.73	52.1	49.85	48.12	44.85
		backward	62.13	61.9	61.6	61.2	60.77	60.42	60.08	59.78	59.73	59.33	58.62
	2	forward	176.7	175.6	173.7	171.2	167.8	163.7	159.3	154.6	149.2	144.3	140.1
		backward	185.5	184.6	183.2	181.7	180.5	179.7	179	178.5	177.5	176.6	175.3
3	1	forward	57.83	57.57	57.08	56.32	55.32	53.93	52.4	50.03	47.62	45.15	42.2
		backward	62.13	61.83	61.33	60.82	60.28	59.87	59.73	59.25	58.92	58.52	58.32
	2	forward	176.7	176.5	176.1	175.7	175.2	174.4	173.5	172.4	171.2	169.8	168.5
		backward	185.5	185.4	185	184.6	184.2	183.7	183.1	182.8	182.4	182	181.7

6. Conclusion

In this work, we studied the influence of an open transverse crack on the vibratory behavior of rotors, a program was developed in MATLAB to generate the Campbell diagram to identify the natural and critical frequencies. After validation of the results of our program with a results founded with a simulation by ANSYS Workbench and other of experimental, we studied the effect of the position and the depth of the crack on the natural and critical frequencies of the rotor, and we noticed that:

- The natural and critical frequencies of a cracked and uncracked rotor have a gap that can be used to identify the presence , depth and position of the crack
- The natural and critical frequencies decrease with the increase of the crack, although this decrease is slight the danger is significant, this decrease is slight in the backward modes by that in forward modes.
- The variation curve of natural and critical frequencies with respect to the position of the crack is similar to the shape of deformation mode.
- More we discretize the rotor the detection of the crack became difficult because the values of experimental result confounds with several values of the numerical results of different cases of position and crack depth.
- Detection of position and depth of crack is not easy and has become very difficult in cases of complex structure rotors.

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