Short-term Account Receivable Securitization Pricing Based on the Cumulative Prospect Theory

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Abstract: *The short-term account receivable securitization presents some characteristic, such as issuing at a discount, absence of repaying ahead of original debtors and lack of reasonable pricing reference rate, therefore the popular asset securitization pricing model is embarrassing, especially repay ahead of schedule model and OAS model. This paper constructs a short-term account receivable securitization pricing model based on the cumulative prospect theory, which derives the optimal price when the utility of investors is maximal, and gives a simulation case. Results show that the model has a single optimal price which only depends on the characteristics of risk appetite and loss aversion of investors given distribution function of market random disturbance and issuing value. Simulation case indicates that optimal price is acceptable so this model can be as a reference method of short-term asset securitization pricing.*

Keywords: *asset securitization, cumulative prospect theory, utility, risk appetite*

1 Introduction

The research of asset securitization mainly focuses on the long-term asset, such as residential mortgage loan, but is indifferent to short-term account receivable. In reality, there are some significant distinctions about the structure of cash flow and risk between the two kinds of asset. Thus, it is critical that whether the popular pricing model can do well or not.

There are three kind of basic pricing model for asset securitization, namely default probability model, prepayment mode and OAS model. Default probability model derive from credit scoring approach and is uppermost to asset pricing. It would got risk adjusted price of asset through estimating the credit default probability of obligator and discounting cash flow. There are lots of literatures about asset securitization in recent year, such as Kau etc (2009), Pinheiro & Savoia (2009), Chang (2011), Fabozzi & Vink (2012)[1-4]. The core of prepayment mode is to estimate prepayment rate of obligator that would impact crash flow structure. Christopoulos etc (2008)、Zhout (2010), Qian (2012) had studied securitization asset pricing by prepayment mode[5-7]. However, probability model is fragile to market circumstances as it excessively relies on historical data. In recent years, OAS model has been widely used in asset pricing. The key of this model is that we would estimate an average discount rate through simulating all possible situation of future rate. Related Articles are Hull & White (2003), Ericsson & Renault(2006), Pan & Singleton(2011)、Liu(2013)[8-12].

The above models are not satisfying to short-term account receivable securitization pricing. The evidences are following. The probability default of short-term asset is easy to be estimated by empirical data because of credit enhancement and simple structure of cash flow. So, the default probability model is too complex to price precisely. In the practice, short-term securities will be issued at discount so that obligors won't prepay debt that means the foundation of prepayment mode is absent. For now, the scale of short-term securitization asset market is puny. Thus, it is difficult to estimate the average discount rate by OAS model since type of instrument is too scarce to get reference rate. In view of the above content, we will construct a short-term account receivable securitization pricing model based on the cumulative prospect theory and payoff structure and give a numerical case

2. Payoff structure and basic model

Defining the payoff structure of short-term account receivable securitization and basic model are important since the former is the justification for basic model that is fundamental to determine the optimal price.

2.1 Payoff structure

According to the process of issuing short-term account receivable securitization, we give a simple payoff structure as shown in fig.1. Generally, the issuer pay the "fee" to the service agencies, such as bank, accounting firm, law firm etc, and determine the "price" to prepare issuing after purchasing short-term account receivable by "cost" from the "holders". Then, the investors buy security by "price" and hold it to maturity. Investors would gain payoff that equal to the gap between face value, said V. and "price" if obligators did not defult. So, the total payoff of issuing short-term account receivable securitization is equal to face value minus "cost" that would be allocated to issuer, service agencies and investors separately.

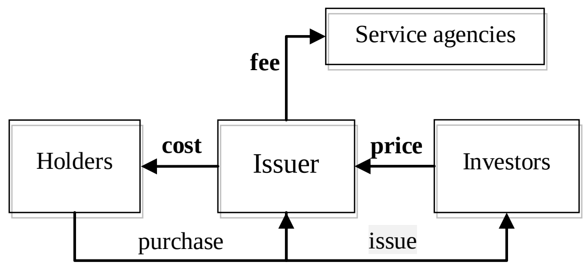

Fig.1 Payoff Structure

We know that the "fee" would be paid to service agencies. According to the process of asset securitization, the "fee" is exogenous in basic model as it is determined by tradition. The "cost" is slightly complex that is a function of the performance probability and average cash deposit ratio of obligators. However, the "cost" is not a key to basic model based on the perspective of investors because it only impacts the gap of face value and "price" indirectly and would not be influenced by investors. So, the "cost" is also exogenous that imply that "cost" would be determined by bargaining of holders and issuer.

2.2 Basic model

How to allocate the residual payoff that equal to "price" minus "cost" and "fee" is crucial to pricing since "fee" and "cost" are exogenous according to the analysis of payoff structure. Further, we concern the ratio that how much the residual payoff would be allocated to investors. Similar as "cost", the ratio allocated to investors would be determined by bargaining of issuer and investors. Three hypotheses are given by following.

(1) Short-term account receivable is issued at a discount, namely investors would gain face value of security that is higher than purchase price; (2)There is no risk of default from issuer since outer credit addition would be present; (3) Security would be held-to-maturity by investors that means no discount and redemption.

Now, we determine $1-\pi$ as the ratio that investors gain from residual payoff rate r and x as payoff of investors from investing short-term account receivable security. r is a function of q, shown as $r = a - bq(a > 0, b > 0)$. Apparently, $\partial r / \partial q < 0$. It indicates that r is monotone decreasing with q decreasing due to the relation between supply and demand. Then, the basic model based on perspective of investors is

$$x = (1-\pi)qr + \theta \tag{1}$$

Among (1), q is the issue amount of securitization asset and θ is stochastic disturbance of market. According to the process of asset securitization, issuer would determine the amount of security in line with specific market condition. Thus, q is exogenous in basic model and $(1-\pi)qr$ is the expectation of residual payoff. Stochastic disturbance θ obey Weibull distribution. Davics & Satchcll(2004) point out that Weibull distribution would describe fluctuation of variable more efficaciously than normal distribution[13].

3. Optimal pricing

We construct the basic model of investors according to the analysis of payoff structure in section 2. In this section, we will derive the optimal pricing model and constraint conditions of short-term s account receivable based on perspective of investors and CPT theory.

3.1 Total utility function

According to the research of Tversky & Kahneman(1993) [14], the function of value and weight of investors are

$$v(x)=\begin{cases}(x-(1-\pi)qr)^{\alpha} & x\ge(1-\pi)qr\\ -\lambda((1-\pi)qr-x)^{\beta} & x<(1-\pi)qr\end{cases} \tag{2}$$

$$\begin{cases}w(p)=e^{-(-Inp)^{\gamma}} & x\ge 0\\ w(1-p)=e^{-(-In(1-p)^{\gamma}} & x<0\end{cases} \tag{3}$$

Among them, α, β, λ and γ are parameter and $p=1-F(x)|_{x>0}$, $1-p=1-F(x)|_{x<0}$ which specific content can be got in Tversky & Kahneman(1993). Let $\theta=\frac{x-(1-\pi)qr}{1-\pi}$ according to (1) and take it into function of Weibull distribution and probability density.

$$x\sim f(x)=\begin{cases}\frac{a_1}{l_1^{a_1}}(\frac{x-(1-\pi)qr}{1-\pi})^{a_1-1}e^{-(\frac{x-(1-\pi)qr}{(1-\pi)l_1})^{a_1}} & x\ge(1-\pi)qr\\ \frac{a_2}{l_2^{a_2}}(\frac{(1-\pi)qr-x}{1-\pi})^{a_2-1}e^{-(\frac{(1-\pi)qr-x}{(1-\pi)l_2})^{a_2}} & x<(1-\pi)qr\end{cases} \tag{4}$$

$$x\sim F(x)=\begin{cases}1-e^{-(\frac{x-(1-\pi)qr}{(1-\pi)l_1})^{a_1}} & x\ge(1-\pi)qr\\ 1-e^{-(\frac{(1-\pi)qr-x}{(1-\pi)l_2})^{a_2}} & x<(1-\pi)qr\end{cases} \tag{5}$$

The total utility $U(x)$ of investing short-term account receivable security based on perspective of investors is

$$U(x)=U^{+}(x)+U^{-}(x)=\int_{(1-\pi)qr}^{\infty}v(x)\frac{\partial w(p)}{\partial p}\frac{\partial p}{\partial x}dx+\int_{-\infty}^{(1-\pi)qr}v(x)\frac{\partial w(1-p)}{\partial(1-p)}\frac{\partial(1-p)}{\partial x}dx \tag{6}$$

Equation (6) indicates that the total utility $U(x)$ is the sum of product of the value function and the weight of each payoff. So, the optimal price means that the total utility of investors is highest at this price, namely

$$P^{*}=\{P\,|\,MAXU(x)\}$$

Firstly, we solve the $U^{+}(x)$. Take equation (1) ~ (5) into (6) and obtain

$$U^{+}(x)=\int_{(1-\pi)qr}^{+\infty}(x-(1-\pi)qr)^{\alpha}\frac{\gamma a_1}{l_1^{a_1\gamma}}(\frac{x-(1-\pi)qr}{(1-\pi)})^{a_1\gamma-1}e^{-(\frac{x-(1-\pi)qr}{(1-\pi)l_1})^{a_1\gamma}}dx$$

Let $y=(\frac{x-(1-\pi)qr}{(1-\pi)l_1})^{a_1\gamma}$ and take into above-mentioned equation. The $U^{+}(x)$ is

$$U^{+}(x)=(1-\pi)^{\alpha+1}l_1^{\alpha}\Gamma(\frac{\alpha}{a_1\gamma}+1) \tag{7}$$

Similarly, we can get the $U^{-}(x)$ as

$$U^{-}(x)=-\lambda(1-\pi)^{\beta+1}l_2^{\beta}\Gamma(\frac{\beta}{a_2\gamma}+1) \tag{8}$$

$\Gamma(.)$ is the gamma function in. So, the total utility function $U(x)$ is

$$U(x)=(1-\pi)^{\alpha+1}l_1^{\alpha}\Gamma(\frac{\alpha}{a_1\gamma}+1)-\lambda(1-\pi)^{\beta+1}l_2^{\beta}\Gamma(\frac{\beta}{a_2\gamma}+1) \tag{9}$$

3.2 Optimal pricing model

Equation (9) is the total utility function of investors who hold the short-term account receivable security. The optimal price P^{*} must satisfy the conditions of $P^{*}=\{P\,|\,MAXU(x)\}$. There is a interesting thing that $U(x)$ is irrelevant to parameters in basic model except π. Furthermore, it is reasonable that α, β, λ , γ and parameters of Weibull distribution and density are stable given specific investors group and market conditions in any period of time. Thus, π is the single variable in equation (9). Let derivative of π equal to 0, namely

$$(\alpha+1)(1-\pi)^{\alpha} l_1^{\alpha}\Gamma(\frac{\alpha}{a_1\gamma}+1)-\lambda(\beta+1)(1-\pi)^{\beta} l_2^{\beta}\Gamma(\frac{\beta}{a_2\gamma}+1)=0$$

The first-order condition of total utility function $U(x)$ can be written as

$$\pi^*=1-\left[\frac{\lambda l_2^{\beta}(\beta_2+1)\Gamma(\frac{\beta}{a_2\gamma}+1)}{l_1^{\alpha}(\alpha+1)\Gamma(\frac{\alpha}{a_1\gamma}+1)}\right]^{\frac{1}{\alpha-\beta}} \tag{10}$$

The necessary condition is $\partial U^2(x')/\partial\pi^2<0$ if total utility function $U(x)$ get maximum value at $\pi=\pi^*$. So, $\partial U^2(x')/\partial\pi^2$ is

$$\frac{\partial U^2(x)}{\partial\pi^2}=-\alpha(\alpha+1)(1-\pi)^{\alpha-1}l_1^{\alpha}\Gamma(\frac{\alpha}{a_1\gamma}+1)+\lambda\beta(\beta+1)(1-\pi)^{\beta-1}l_2^{\beta}\Gamma(\frac{\beta}{a_2\gamma}+1) \tag{11}$$

Obviously, $\pi\in[0,1]$ and the value of $\partial U^2(x')/\partial\pi^2$ would be determined by α, β, λ , γ and parameters of Weibull distribution and density. Now, we give these parameters a range according to the studies of other scholars. We determine the value of parameters of Weibull distribution and density based on the studie of Davies & Satchell (2004) that point out $l_1=l_2=0.037$, $a_1=1.268$ and $a_2=1.087$ [15]. Tversky & Kahneman(1993) give a range to value of α, β, λ , γ that is $0<\alpha<1$, $0<\beta<1$, $\lambda>1$ and $0<\gamma<1$ [14]. Following the seed literature of Tversky & Kahneman, some scholars estimate the parameters of CPT theory, such as Malevergne & Sornette (2001), and find that the estimates are uniform to Tversky & Kahneman(1993) [16]. So, the ranges of parameter values conform to the following system of inequalities in this paper.

$$\begin{cases} l_1=l_2\in(0,1);1<a_2\le a_1 \\ 0<\alpha<\beta<1;\lambda>1 \\ 0<\gamma<1 \end{cases} \tag{12}$$

Take equation (12) into (11) and get

$$\begin{cases} \alpha(\alpha+1)<\beta(\beta+1) \\ (1-\pi)^{\alpha-1}<(1-\pi)^{\beta-1} \\ l_1^{\alpha}>l_2^{\beta};l_1^{\alpha}<\lambda_2 l_2^{\beta} \\ \frac{\alpha}{a_1\gamma}<\frac{\beta}{a_2\gamma};\Gamma(\frac{\alpha}{a_1\gamma}+1)<\Gamma(\frac{\beta}{a_2\gamma}+1) \end{cases} \tag{13}$$

According to the (13), $\partial U^2(x')/\partial\pi^2$ is negative that means $U(x)$ would get maximum value at π^*. Taking equation (10) into (1), the optimal expected payoff of investors is

$$E(x^*)=E[(1-\pi^*)(qr+\theta)]=(1-\pi^*)qr \tag{14}$$

Based on the payoff structure and hypotheses of basic model in section 2 , we know $E(x)=(V-P)q$. So, the optimal pricing model of investors is $P^*=V-E(x^*)q^{-1}$, namely

$$P^*=V-V(a-bq)\left[\frac{\lambda l_2^{\beta}(\beta+1)\Gamma(\frac{\beta}{a_2\gamma}+1)}{l_1^{\alpha}(\alpha+1)\Gamma(\frac{\alpha}{a_1\gamma}+1)}\right]^{\frac{1}{\alpha-\beta}} \tag{15}$$

There would be a unique value of optimal price given q if risk characteristics of investors, parameter a, b and Weibull distribution are exogenous.

3.3 Constraint condition

We have provided the optimal pricing model when the total utility of investors is maximal based on CPT theory. But the work is not all as investors are rational in reality, namely, they would compare the payoff of short-term account receivable security with others and choose the higher.

Determining r_s and r_l are the expected yield of short-term account receivable security and others respectively with the identical deadline. When the total utility of investors is maximal, r_s is

$$r_s = \frac{x^* q^{-1} V}{P^*} = \frac{(1-\pi^*)r}{(1+r-r\pi^*)} \tag{16}$$

Investors would hold the short-term account receivable security to maturity if they consider that the expected yield of short-term account receivable security is no less than other assets. So, equation (16) would satisfy the constraint condition $r_s \geq r_l$. In combination with (16), we get

$$(1-\pi^*)(1+r-r\pi^*)^{-1} r \geq r_l \tag{17}$$

It is obviously that $1+r-r\pi^* > 0$ and $(1-\pi^*) > 0$, then the equation (16) is equivalent to

$$-b(1+r_l\pi^* - \pi^*)q \geq -a(1-\pi^*) - ar_l\pi^* + r_l \tag{18}$$

Similarly, there are $1+r_l\pi^* - \pi^* > 0, b > 0$ and we know that $-b(1+r_l\pi^* - \pi^*) < 0$. Since there is a one-to-one correspondent relationship between optimal price P^* and q, the constraint condition of optimal pricing model is following

$$0 \leq q \leq \frac{a(1-\pi^*) + ar_l\pi^* - r_l}{b(1+r_l\pi^* - \pi^*)} \tag{19}$$

4. Simulation and numerical case

In section 3, we have constructed the optimal pricing model of investors based on CPT theory and derived the constraint condition. It is interesting and significant that how the optimal price would change if the parameters changed. So, we give some simulated analysis about the relationship between the optimal price and parameters, such as α, β, λ, π^* and q, and a numerical case in this section.

4.1Simulation

We would be interesting in three kinds of simulation relationship between the optimal price and independent variables based on equation (16) if the exogenous variable a, b, V and Weibull distribution and density functions were given. Firstly, we focus on how the optimal price will vary with different α and β since they are stable to specific investors and crucial to sharing coefficient π. Secondly, the relationship between the optimal price and loss aversion coefficient λ would be taken into our vision by the uniform reason as α and β. Thirdly, we concern the combined impaction on optimal price in line with π and q varying. The simulation results are shown as fig.2 to 4 if the values of exogenous variables meet system of inequalities (12) and are indifferent to constraint condition

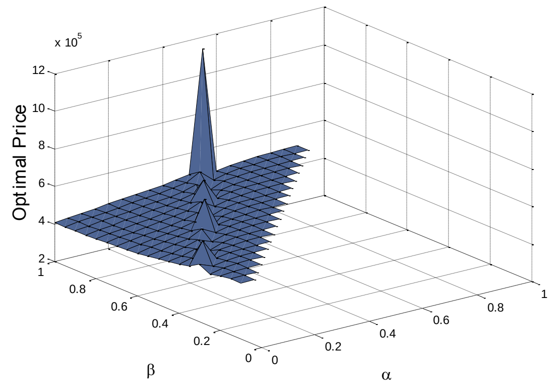

Fig. 2: Relationship between P^* and α , β

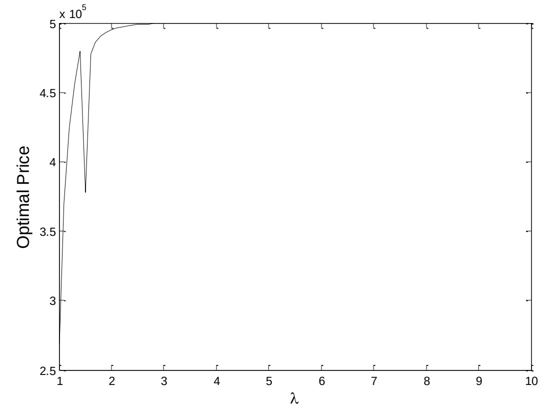

Fig. 3: Relationship between P^* and λ

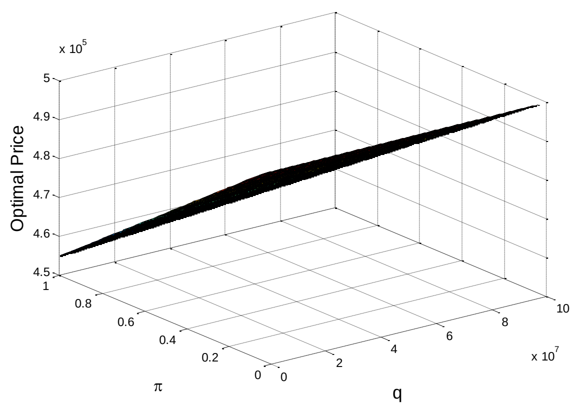

Fig. 4: Relationship between P^* and q , π

The optimal price is insensitive to α or β varying in fig.2. We find that the optimal price will increase slightly except for the point that is around $\alpha = 0.6$ and $\beta = 1$ when α increase or β decrease. Although variation tendency is equivocal around $\lambda = 1.5$, the optimal price will increase as similar to α or β when loss aversion coefficient λ increase in fig.3. The above simulation conclusions imply that the optimal price is positive to the degree of risk aversion of investors. There are two arguments to interpret the relationship between the optimal price and π or q in fig.4. One is that the remaining yield r would decrease alone with q increasing because of the outcome of supply-demand relationship. The other is that the proportion of remaining yield r shared by investors would decrease if π increased. Therefore, the optimal price would increase accompanied by r decline.

4.2 Numerical Case

We assume that a financial institution would issue short-term account receivable security with three months term and face value of 500000 dollars. The opportunity cost of holding short-term account receivable security is $r_l = 0.0175$, which is the mean of other similar finance products with same term. According to the Davies & Satchell (2004), Tversky & Kahneman(1993),Malevergne & Sornette (2001) and Yong D.T. etc (2005), the specific parameter values in equation (16) is shown by Tab.1.

TABLE I: Parameter Hypothesis

$l_1 = l_2$	a_1	a_2	α	β	γ	λ	a	b
0.037	1.27	1.09	0.4	0.7	0.59	2.25	0.09	10^{-8}

The first-order condition of $U(x)$ is $\pi^* = 0.09112$ and we obtain the value of q according to the constraint condition, namely $q \in [0, 8.224 \times 10^7]$. Based on computation of the above, the optimal price P^* is from 0 to

4.6274×10^5 corresponding to $q\in[0,8.224\times10^7]$. The relationship is shown by fig. 5. The optimal price is intuitively acceptable since the P^* is less than face value.

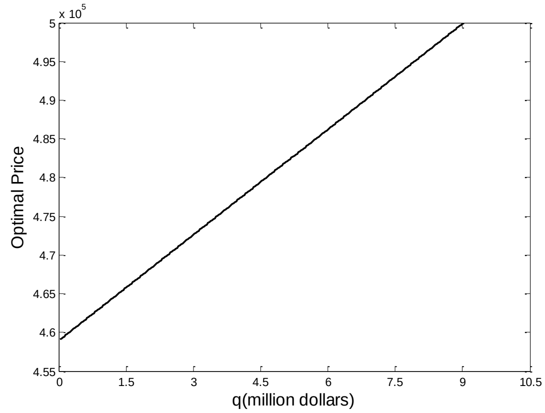

Fig. 5: Result of optimal price Simulation

5. Conclusions

This paper constructs a short-term account receivable securitization pricing model based on cumulative prospect theory. Different from repay ahead of schedule model and OAS model, our model would derive the optimal price according to the payoff structure when the utility of investors is maximal. Thus, it is more rational to characteristics of cash flow and risk structure of short-term account receivable security.

We find that the optimal price of short-term account receivable security would vary if the sharing coefficient or issuing value changed. Thus, the risk attitude characteristics of investors and market scale of issuing value are dominant to optimal price in our model given the value of other parameters, such as a, b, V and Weibull distribution and density functions. The optimal price would increase slightly if the risk aversion level of investors enhanced which is denoted by α, β and λ based on CPT theory. So, we consider the optimal price is insensitive to risk characteristics of investors. The residual yield r is negative to issuing value q because of effect of supply and demand in short-term account receivable market. The impact of issuing value changing is more violent than risk attitude characteristics of investors.

This paper imply that the optimal price is determined primarily only by issuing value in specific short-term account receivable market within a short term which means that risk attitude characteristics of investors is steady.

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